Stable Mergers and Cartels Involving Asymmetric Firms

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Abstract

The endogenous formation of coalitions involving asymmetric firms and their stability are analyzed as a function of differences in efficiency and of the fixed cost of production. Results are derived for cartels as well as for mergers. Players have constant but different marginal costs of production and no rule of profit sharing is fixed. The analysis is illustrated for a specific path of collusion. Finally welfare effects are studied and some conclusions are drawn for antitrust policy.

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1. Introduction

The subject of coalition formation has received in recent years a growing attention in economic literature, accompanying the increasing importance of merging as a competitive strategy in oligopolistic industries. After a period in which the evolution of the industry has been analyzed mostly in terms of entry and exit, following major developments in noncooperative game theory, attention seems now to be turning to mergers and acquisitions as another source of industry evolution, giving rise to a new interest in cooperative game theory.[1] Still, as this new literature places great emphasis on the "noncooperative foundations for cooperative behavior", links between both …elds of Game Theory are now better understood than before.

A signi…cant part of merger operations occurs for …nancial reasons (speculative motives); however, strategic motives such as increasing e¢ciency or enhancing monopoly power are also at the origin of many mergers.

Collusive behavior may take several forms: …rst it may be explicit or implicit. The implicit form is generally known as tacit collusion and requires a dynamic context to be implementable. The explicit form of collusive behavior - mergers, cartels and joint ventures - can be more or less complete. Mergers are the most complete type of collusive agreement: the separate entities of the constituent …rms disappear and start acting as a single unit thereafter. At the other extreme lie joint ventures, which are designed for …rms to bene…t from scale economies while remaining separate entities.

In this paper we deal with horizontal mergers (i.e., mergers among …rms in the same product market)[2] and cartels, and analyze the role played by e¢ciency di¤erences in their motivation and stability. This issue is still far from being completely and satisfactorily explored in the literature, due to the di¢culty in handling asymmetry. In fact, symmetry between players is usually assumed and when heterogeneity is allowed, a …xed rule of payo¤ division is postulated (e.g. Farrell and Scotchmer(1988) and Bloch(1996)). We do not …x any rule of pro…t sharing, rather leaving it for the bargaining process between joining …rms, which is assumed to be costless.

We consider that an agreement is reached whenever it is pro…table for the …rms

[1] The problem of the choice between internal growth (entry by building) and external growth (entry by buying) has been addressed for example by Gilbert and Newbery(1992).

[2] Mergers are often categorized as horizontal, vertical, or conglomerate. Horizontal mergers take place between …rms in the same line of business; in a vertical merger the buyer expands backward or forward; conglomerate mergers involve companies in unrelated lines of business. See for instance Scherer and Ross(1990) and Brealey and Myers(1988) for some historical perspective on these various types of mergers and for some examples.

involved. By imposing the condition that joint profits should be greater than or equal to the sum of the ex ante profits of the firms involved, we guarantee that no firm loses when it decides to join with another; we assure that, in the absence of bargaining costs, there can be some division of profits between them which makes it worthwhile for both to join, without fixing an *a priori* rule of sharing.[3] We are therefore allowing for side payments, otherwise the heterogeneity of firms would make it more difficult to find admissible agreements (i.e., individually rational agreements) and to sustain them.[4]

In turn, for some arrangement to be stable this kind of incentive to merge further (broaden the coalition) must not exist (external stability). These two conditions - profitability and external stability - together with a division of the coalitional profit are sufficient to determine equilibrium mergers. For equilibrium cartels a condition for internal stability (that no firm benefits from leaving the agreement) must also be set. Our stability analysis proceeds stepwise, from a given partition to an "adjacent" partition (that is, with one player less or one player more); hence the aggregation movements we consider involve just two firms or groups of firms and the disaggregation movements give rise to just one additional player.

The treatments which are closest to ours, in the sense of also dealing with heterogeneity of the agents, are Farrell and Shapiro(1990a), who consider quite general cost functions but do not model the merging process, Faulí-Oller(1995), who restricts asymmetry to the existence of two different types of players, the efficient and the inefficient, and Ray and Vohra(1996), who study the endogenous formation of coalitions in the symmetric case as well as in the general case of asymmetry. We model asymmetry by introducing a parameter α which stands for the (constant) marginal cost difference between firms. The analysis on the incentives to form coalitions and their stability is then performed as a function of α and of a parameter f representing the fixed cost of operating a firm.

Following the positive analysis we study the consequences of the firms' agreements on social welfare and derive some conclusions about the social desirability of different concentration movements. It is shown that for small efficiency differences no merger is good in terms of total welfare in the absence of significant cost savings, whereas for large α the optimal merger is welfare-enhancing even if there are no savings in fixed cost ($f = 0$), and, as expected, involves the absorption of a very inefficient firm. The relationship between changes in market concentra-

[3] Many ways of modeling the process of the formation of coalitions have been proposed in the literature. Ours is the simplest: the coalition emerges whenever it is profitable.

[4] The side payments which we allow for are within coalitions. Transfers between coalitions are not allowed (so a first best cannot be achieved).

tion and changes in economic welfare is also analyzed and conclusions are drawn for approval rules. An antitrust policy based on concentration measures may be misleading, but the best movement in terms of social welfare and the one which produces the smallest increase in concentration are the same in most of the cases.

The analysis of this paper can be extrapolated to other social situations in which individuals differing by an observable characteristic (and having a fixed cost of performing some activity) may want to form coalitions. Indeed, the subject of coalition formation has already been applied to different situations such as trading structures in bilateral oligopolies (Bloch and Ghosal(1997)), the formation of associations including R&D joint ventures or the adoption of a common standard (Bloch(1995)), the formation of syndicates (Guesnerie(1977) and Greenberg(1979)), the possibility of mergers between domestic and foreign firms (Horn and Persson(1997b)), trade agreements (Macho et al (1994)), customs unions (Yi(1996)), international agreements for the protection of the environment (Carraro and Siniscalco(1993)) and voting games (Peleg(1984)), among others.

The paper is organized as follows. Section 2 presents the game-theoretic solution concepts usually employed for analyzing the formation and stability of coalitions and also provides a revision of the main topics usually addressed in the Industrial Organization literature on mergers and cartels. Section 3 presents the analytical framework and an illustration for the case of three firms. Section 4 generalizes for the case of n firms and establishes the main results of the paper. Section 5 contains the welfare analysis and section 6 concludes. Formal proofs and details are given in the Appendix.

2. The GT tools and the IO literature

In this section we briefly present the game-theoretic tools usually employed to study the subject of coalition formation and go through the main literature on mergers and cartels, from an Industrial Organization perspective.

2.1. The GT tools

The analysis of coalition formation within the framework of cooperative Game Theory makes use of cooperative solution concepts such as the core (for example, Aumann(1967) and an application to trade agreements in Macho et al (1994)), the von Neumann and Morgenstern stable sets (Espinosa and Iñarra(1995)) or the Shapley value (Hart and Kurz(1983) use a variant of the Shapley value, the "coalition structure value").

This cooperative framework, however, ignores externalities among coalitions and is not suitable to describe the formation of coalitions as a noncooperative process. It seems reasonable to believe that there are spillovers (externalities, either positive or negative) to the other players when some decide to join. Hence, in recent years many authors have resorted to noncooperative solution concepts to determine the equilibrium number, size and composition of coalitions,[5] especially considering games in extensive form. Examples of this approach are Bloch(1996), Kamien and Zang(1990, 1991 and 1993), Nilssen and Sorgard(1998), Yi and Shin(1995) and Faulí-Oller(1995).

Extensive-form games capture situations in which players announce sequentially their desire to take part in a coalition (sequential games of coalition formation) and are especially appropriate to capture forward-looking behavior by agents. These models with "farsighted" players, in which decisions are based on the …nal outcome, that is, considering all the chain reactions to the present move until the end, are one of the …elds that currently deserve more interest (see for instance Ray and Vohra(1996) and Espinosa(1996), this latter as an application of the largest consistent set notion of Chwe(1994)).[6]

Within the framework of strategic-form games, the most important solution concepts are extensions of the Nash equilibrium; these are appropriate to games in which all players simultaneously announce their decision to cooperate (simultaneous games of coalition formation).

In many situations in real life it is not possible to sign binding agreements and therefore agreements must rely on their self-enforceability. The Nash solution is a necessary condition for self-enforceability, but is not su¢cient: in fact, it is usually possible for players to make some arrangements (form coalitions) which are mutually bene…cial, provided that all the others do not change their strategies (take the actions of the complement as …xed). The simple Nash solution excludes this situation, but the concepts of strong Nash equilibrium (SNE) and coalition-proof Nash equilibrium (CPNE) contemplate it. These re…nements to the Nash solution limit the set of equilibria (usually very large) and thus allow more accurate prediction. Since they are of a cooperative nature we can say that simultaneous games are actually on the border line between cooperative and noncooperative game theory.[7]

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[5] This distinction between cooperative and noncooperative approaches corresponds to the distinction between the two types of representation of games, in coalitional (or characteristic) function form and in partition function form.

[6] The forward-looking behavior induces a higher degree of cooperation because …rms tend to resist incentives to cheat, anticipating the consequences.

[7] Yi and Shin(1995) separate simultaneous games of coalition formation into two categories: open membership games, where players are free to join or leave any coalition, and exclusive membership games, where the members of the coalition are allowed to deny membership to outsider …rms.

According to the notion of SNE (Aumann(1959)) an equilibrium agreement must be immune to deviations by every conceivable coalition, while according to the notion of CPNE (Bernheim et al (1987) and Bernheim and Whinston(1987)) an equilibrium agreement must be immune to deviations by every conceivable coalition which are also immune to further deviations by members of the deviating group (consistency requirement; notice that it embeds some forward-looking behavior, though players see only one step ahead).[8] That is, some deviations which would invalidate a SNE do not invalidate a CPNE, because they are not valid deviations and therefore a CPNE has more chance to "survive" deviating movements.[9] In this sense the SNE is a stronger concept and more difficult to find.

More recently there have appeared cooperative tools that take externalities into account. The equilibrium binding agreements of Ray and Vohra(1997) are the "parallel" of CPNE, since they rule out coalitional deviations that are not themselves immune to further deviations by subcoalitions.

2.2. The IO literature

According to the assumptions made and to the solution concept employed, results in the literature have ranged from the impossibility of merging up to monopoly (v.g., Kamien and Zang(1990 and 1991), provided that the industry has sufficiently numerous firms) to models which predict the possible complete monopolization of the industry in the absence of regulatory devices, as for example in Salant et al (1983) (henceforth sometimes referred to simply as SSR).

In SSR's paper it is shown that in a Cournot setting exogenous mergers may be unprofitable for the firms involved, therefore creating few incentives to join: the reason is that the new firm tends to reduce quantity (because competition is made less aggressive by the merger), causing an increase in the quantity produced by

[8] The fact that only deviations by members of the deviating group are considered in the concept of CPNE is still a limitation. The study of abstract stable sets in which the dominance relation allows for the possibility of a subset T of a deviating coalition S attracting a coalition $Q \subseteq N \setminus S$ (where N is the universe of players) in order to jointly further deviate has given rise to some equilibrium concepts, depending on the information structure. If previous agreements are common knowledge and under the assumption that all other players will stick to the announced tuple of strategies, the appropriate solution is the one resulting from "coalitional contingent threats" (Greenberg(1990), pgs.102-6): if some coalition declares that its members will adopt a different strategy profile any other coalition may respond in the same way to the revised proposal and the process continues until an equilibrium is found.

[9] The concept of CPNE can also be applied to extensive form games, giving rise to perfect coalition-proof Nash equilibria (e.g., Matutes and Padilla(1994) for an application of perfect CPNE in pure strategies to the formation of shared ATM networks).

outsiders. In the absence of …xed costs and under the assumptions of symmetry and homogeneity of the product this is the sole e¤ect; the joint pro…t is thus smaller than the sum of the pre-merger pro…ts of the …rms involved and so there is no incentive to merge. Mergers must then be motivated by monopoly power alone and the authors show that with linear demand it is necessary that at least 80% of the …rms in the industry merge to pro…tably exploit market power (minimal pro…table coalition size). Some regulatory device that prevents more than $\frac{4}{5}$ of the industry from colluding therefore makes full monopolization impossible. Cheung(1992) reduces this minimal market share to 50% by allowing for any demand satisfying second-order conditions.

The result on the private unpro…tability of mergers can be reversed for example if the savings in …xed cost implied by the operation are su¢ciently important. When …rms are not equally e¢cient, as in our paper, there is an additional e¤ect that may cause this merger to be pro…table even in the absence of …xed costs, the switch of production from the ine¢cient …rm to the e¢cient one. A similar idea is present in the criticism of Perry and Porter(1985) to SSR: the new cost function must be di¤erent from the initial one, since as a result of merging, the …rm now has access to a technology which may be strictly more e¢cient than the one before; the price increase can be su¢cient to compensate for the decline in production, therefore making the merger pro…table.

With Bertrand competition mergers are always bene…cial (because the reaction of outsiders reinforces the price increase due to merger) and the more …rms involved, the more pro…table they are (Deneckere and Davidson(1985)). Other attempts to reverse the results by SSR on the private unpro…tability of merging include relaxing the assumption of homogeneity of the product. The existence of product di¤erentiation reinforces the results obtained under price competition and allows mergers to be privately pro…table even under quantity competition as long as the degree of di¤erentiation is su¢ciently high (Granero(1997)).

The abandonment of the assumption that the newly created …rm remains a Cournot player, by allowing it to behave as a Stackelberg leader if its size is large enough,[10] also reverts SSR's results (again Granero(1997) and also Daughety(1990)). Faulí-Oller(1996), in turn, has considered a more general demand function than that employed by SSR and has studied the e¤ects of its degree of concavity on the pro…tability of the agreement; Gaudet and Salant(1991) have also reached the possibility of pro…table mergers under general demand and cost functions (though equal for all …rms). In the current paper the symmetry as-

[10] See Sadanant and Sadanant(1996) for the grounds of approximating "the behavior of a dominant …rm with a …nite fringe (...) by Stackelberg equilibrium". Sha¤er(1995) also considers Cournot behavior within the fringe and leader behavior by the cartel.

sumption is relaxed and a reversion of SSR's results is also obtained: mergers can be privately pro…table even with no …xed cost savings and under all the other assumptions of SSR, provided …rms are su¢ciently asymmetric.

The incentive to free-ride (let the others merge and stay out of those groups), also known as the "hold-up" problem, subsists in all these variations, as outsiders earn more than …rms participating in the merger.[11] The recognition of the free-riding problem was …rst made by Stigler(1950), who stated that the promoter of a merger could receive every encouragement from the other …rms but participation. Szidarovszki and Yakowitz(1982) show that if some …rms form a cooperative group their joint pro…t can fall below that of the non-cooperative situation, whereas the pro…t of the non-cooperating …rms does not decrease. The same kind of idea is present in the price-leadership model of d'Aspremont et al(1983), in which fringe …rms enjoy higher pro…ts than members of the dominant cartel.[12] These are clear examples of positive spillovers in games of coalition formation (coalition members provide some type of public good, which external …rms enjoy without supporting its cost). The incentive to free-ride is at the origin of the potential instability of coalitions.

Since the work of Hart and Kurz(1983), and bearing in mind the unpro…tability characteristic of some exogenous mergers, as detected by Salant et al(1983), several papers have tried to model the process of coalition formation in an endogenous way, trying to predict which mergers will occur in a given situation. With a purpose similar to ours but assuming homogeneity of …rms, Rajan(1989) derives the endogenous formation of coalitions as a function of some relevant market characteristics. Kamien and Zang(1990 and 1991) use an allocation scheme based on the bids each …rm makes for all the others and on the asking price each …rm announces for itself to determine which acquisitions will take place. Ray and Vohra(1996) suggest a bargaining process in which players are farsighted and therefore care about the end results of their moves to select which proposals will be made and accepted. This is actually the line of research that is currently receiving more attention.

The desirability of mergers in terms of welfare is ambiguous, due to the trade-o¤ between e¢ciency gains and reduced competition (there are gains ac-

[11] In our variation (asymmetry) it is not clear whether this incentive subsists or not, because the payo¤ for each member of the agreement is unknown (remember that we do not …x any rule of division) and thus cannot be compared with the payo¤ of the outside …rms.

[12] This model is the simplest example of an open membership game. It makes use of the notions of internal and external stability, which we also employ in the present paper. The incentive to free-ride is not present in d'Aspremont et al(1983)'s model if the fall in price due to increased competition as one …rm defects gives rise to a fall in pro…t that o¤sets the advantage of joining the competitive fringe (Donsimoni et al(1986)).

cruing to increased productive e¢ciency, and losses due to allocative ine¢ciency - Williamson(1968)). Horizontal mergers have undesirable e¤ects on price-cost margins, both through the concentration index and through the conjectural variation (refer to the Lerner index of Cowling and Waterson(1976)). However they also have bene…cial e¤ects on cost, as the e¢cient …rms increase their market shares at the expense of the ine¢cient ones (unless the X-ine¢ciency of Leibenstein(1966) dominates). So, total industry pro…t and per …rm pro…t tend to increase.

In the seminal work of Szidarovszki and Yakowitz(1982) it is shown that, under certain assumptions for the unit price function and for the (di¤erent) cost functions of the …rms, cooperative grouping implies a decrease in production levels. This is true in most models of merging and is at the origin of the negative e¤ect implied on consumers' welfare. Concerning total welfare SSR prove that unpro…table exogenous mergers may still be socially desirable, due to e¢ciency gains resulting from savings in …xed cost. On the other hand when mergers are modeled endogenously (Kamien and Zang(1990), v.g.) socially bene…cial mergers that are unpro…table for the …rms involved will not take place and the socially undesirable mergers that are privately bad will also fail to occur in the equilibrium, and therefore need not be of concern.

It is generally believed that horizontal mergers, by reducing the number of …rms, facilitate tacit collusion (see for example Osborne(1976), who considers the bene…ts of a smaller number of …rms for the internal stability of a cartel in terms of making it easier to share the pro…ts and to detect and deter cheating): in this sense they would hurt welfare. This is the impact on the Lerner index via the conjectured response of the rivals. Davidson and Deneckere(1984), however, pointed out the fact that mergers increase the pro…ts earned at the threat point (the equilibrium to which the game reverts if cheating is detected), thereby hindering tacit collusion and potentially raising welfare.

As regards the relationship between concentration and welfare, Farrell and Shapiro(1990a) show that an increase in the Her…ndahl index may not necessarily be welfare reducing and provide su¢cient conditions for pro…table mergers to raise welfare (analyzing the external e¤ects of the merger on the rival …rms and on consumers); these conditions, however, may no longer be su¢cient when mergers are interdependent, in the sense that the realization of some merger today may in‡uence the occurrence of mergers tomorrow, as shown by Nilssen and Sorgard(1998). A positive relationship between welfare e¤ects and level of concentration of the non-participating …rms was found by McAfee and Williams(1992), who also proved that mergers which increase the dimension of the largest …rm or create a new largest …rm reduce welfare.

A common feature of most coalition formation models developed up to now is the lack of heterogeneity of …rms. When di¤erences among …rms are allowed when modeling the merging process, a …xed rule of payo¤ division is assumed (Farrell and Scotchmer(1988) and Bloch(1996)), which highly conditions the results obtained. In this paper we trs o is o s)a peol hsuh allocation of outputs across facilities). Every firm observes the merging activity among the others and knows exactly how efficient its rivals are.

For the sake of simplicity we consider that there is no entry in nor exit from this industry, so the number of players is fixed apart from changes due to joining or separating movements. In an industry with n firms there are $2^n - 1$ possible coalitions. The number of partitions is obviously smaller and the number of k-firm oligopolies even smaller ($1 \leq k \leq n$), though all of them are of course very large. The formulas for the total number of partitions and for the total number of oligopolies with k firms can be found in the Appendix. Notice that in a symmetric industry all k-firm partitions yield the same level of concentration, in other words, it is indifferent which firms join; however if firms are not equally efficient there are different levels of concentration associated with the various k-firm oligopolies.

3.1.2. Payoffs

The payoffs are the profits each firm receives in every given market structure, after competing in quantities. The good is homogeneous, demand is linear and there is no uncertainty about demand conditions. Firms that merge or form a cartel remain Cournot players after the operation. For firms belonging to a group, payoffs are not clearly defined, but depend on the way the members agree to share the common profit, which we do not fix.

3.1.3. Strategies and equilibrium

Given that the rest of the market is unchanged, coalitions $C_1 \neq \emptyset$ and $C_2 \neq \emptyset$ will join and form $C = C_1 \cup C_2$ if and only if

$$\pi_C - f \geq \pi_{C_1} - f + \pi_{C_2} - f$$

where, abusing notation, π_C denotes the variable profit of coalition C given the existing partition of the rest of the market (profitability condition).[14]

The new coalition $C \neq \emptyset$ is externally stable if and only if there exists no profitable way of broadening it (given that the rest of the market is unchanged): for all $C' \subset \overline{C}; C' \neq \emptyset$

$$\pi_{C \cup C'} - f \leq \pi_C - f + \pi_{C'} - f$$

gain to the low-cost firm from the merger is the elimination of the high-cost firm as a Cournot rival".

[14] All notation employed shall be understood in this context: given the existing partition of the rest of the players.

where $\overline{C}$ denotes the complement of C in the universe of players.

It is <u>internally stable</u> if and only if there exists no subcoalition of it which receives a higher payoff by acting alone than by staying in (given that the rest of the market is unchanged): for all $C'' \subset C; C'' \neq \emptyset$;

$$\pi_{C''}|_{C \backslash C''} - f \leq \sum_{i \in C''} (\pi_{(i)}|_C - f)$$

$C \backslash C''$ denotes the complement of C'' in C and the use of $\pi_{C''}|_{C \backslash C''}$ is an abuse of notation that reinforces that coalition C'' is competing against $C \backslash C''$. Indeed, it is already clear that every coalitional profit is defined given the partition of the rest of the market: we are stressing that coalition C'' was taken out of coalition C. This abuse of notation is employed in some other expressions throughout the paper. In the right hand side of the inequality the profit of every $i \in C''$ is denoted by $\pi_{(i)}|_C$, meaning that coalition C is still "complete", and where the use of parenthesis is intended to capture the notion that firm i's profit is unknown as long as the firm belongs to a coalition, since we do not fix any rule of payoff division.

It is assumed that joining and separating are costless movements.[15] For the purpose of our analysis we will consider broadening movements to include just one player more and also separating movements of just one player (C' and C'' are singletons). This corresponds to most of the joining and separating movements actually observed.

Trivially the grand coalition is externally stable, whereas the totally dispersed market structure is internally stable.

We consider two different types of agreements: <u>cartels</u>, in which a number of independent firms join to make price or output decisions (in our case, output) and that suffers from each participant having an incentive to break away and join the competitive fringe, and <u>mergers</u>, the ultimate form of collusion in which firms lose their separate entities. We assume that the main difference between cartels and mergers is that in the first case firms are free to exit the agreement, whereas in the second case they are committed to cooperation (mergers are irreversible, once an agreement is reached it cannot be broken).[16] For the stability analysis

[15] In particular, we consider that the buyer does not pay any premium for the selling firm over its value as a separate entity (represented either by its market value, or by its market value plus the valuation expected by investors following the acquisition).

[16] Both problems become equivalent if we allow for divestitures or for the divisions of the firm that came out of the merger to spin off into several firms and become independent entities at any time (for examples of spinoffs and explanations of motives see, for instance, Habib et al (1997)),

of profitable cartels we thus have to check both internal and external stability, but for the stability analysis of profitable mergers we have to check only external stability.

Our stability analysis is stepwise: we restrict attention to one concentration movement at a time (two firms or groups of firms joining) and we also analyze only one movement of exit from a group at a time. This means, for example, that the simultaneous exit of two firms, each on its own, must be analyzed first as the exit of one and then, in the resulting market structure, as the exit of the other one, otherwise we would be analyzing deviations by more than one coalition of players at the same time. As a consequence, we can only proceed from an oligopoly with k operating firms to an "adjacent" oligopoly, with $k - 1$ or $k + 1$ operating firms (corresponding to external instability and internal instability of the coalition(s) that constitute the k-firm oligopoly, respectively).

The move from an oligopoly with k operating firms to another oligopoly with k operating firms cannot be studied within our framework. As stated before (see section 2.1), a solution concept based on "coalitional contingent threats" would be required.

An equilibrium is defined by a stable partition, according to the definitions given above, and by the corresponding payoff division within its coalition(s).

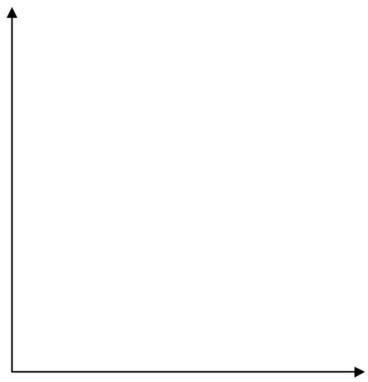
By setting $f = 0$ the results obtained can be identified with an industry with no fixed cost of production, in which case the main parameter driving coalitions (apart from the search for market power) is α; on the other hand, if α is set to zero, then the savings in fixed cost stand as the main determinant of the joining process (idem), as in the work of Espinosa and Iñarra(1995). When α is strictly greater than zero our analysis is an extension of the previous literature on the profitability and stability of coalitions to the case in which firms are not equally efficient.

3.2. The case of $n = 3$

Consider an industry with 3 firms. Firm 1 (the most efficient) has a constant marginal cost of production of $c - \alpha$, firm 2 of c and firm 3 (the least efficient) of $c + \alpha$. There is a fixed cost equal to f. Demand is linear: $P = a - Q$ where Q is total production of the homogeneous good by the industry, $a > c > \alpha$.

With $n = 3$ there are three possible oligopolistic structures: monopoly (the grand coalition, represented either by M or by $\{1,2,3\}$), duopoly and triopoly

or, alternatively, if the cartel has perfect enforcement. The occurence of divestitures is often motivated by the threat of further mergers: selling the main object of the takeover bid may induce the proposed acquirer to drop its bid.



4. The general case of n firms

In this section we present the main results that can be obtained concerning the profitability and stability of coalitions under a general framework; these results are then particularized for a specific collusion path.

The model is as described in subsection 3.1. The game is two-stage: firms form (or not) coalitions and then the resulting groups compete in quantities. For all n firms to be in the market we have to set some prior conditions on the values of α and f, in order to assure positive quantities: these conditions derive directly from imposing $q_n^n > 0$ (where the subscript refers to the n-th firm and the superscript signals an n-firm oligopoly) and $\pi_n^n - f > 0$ respectively (π denotes the variable profit). The first condition gives rise to the restriction on α: $\alpha < \bar{\alpha}(\gamma; n)$ and the second condition gives rise to the restriction on f: $f < \bar{f}(\gamma; \alpha; n) = \pi_n^n$. (See the Appendix for the formal derivations).

4.1. Main results

Having a completely dispersed market, in which all n firms operate separately, we will analyze its stability in terms of the parameters α and f and study the relative profitability of the various two-firm coalitions that transform the market in an $(n-1)$-firm oligopoly. We analyze then the stability of these agreements (by adding to the condition for internal stability the condition for external stability) and try to generalize to broader coalitions, and thus more concentrated market structures.

4.1.1. Two-firm agreements

The n-firm oligopoly is unstable whenever it is profitable for any firms i and j to merge,[19] that is, whenever

$$\pi_{ij}^{n-1} - f \geq \pi_i^n - f + \pi_j^n - f$$

This condition says that in the assumed absence of bargaining costs there is at least one way of sharing the joint profit of i and j such that each firm is better off by joining than by staying alone (the merger represents a Pareto improvement for i and j). It is thus clear that the condition for external instability of the firms i and j) coincides with the condition for the internal stability of coalition $\{i, j\}$. Hence profitability and internal stability are equivalent for two-member coalitions and so it is indifferent whether we are focusing on cartels or on mergers (because internal stability is always met for profitable agreements).

We assume that firm i is more efficient than firm j, so its marginal cost prevails after merging has taken place. This will imply in our notation $j > i$. Solving the above expression for f we get

$$f \geq f_{ij} = \pi_i^n + \pi_j^n - \pi_{ij}^{n-1}$$

For values of f above f_{ij} the n-firm oligopoly is not stable. This boundary is a function of the usual parameters (γ, α and n) and also, now, of i and j (that are discrete variables, just as n) (see the Appendix for the complete expression). The minimum of f_{ij} over i and j gives the upper bound for the stability of the n-firm oligopoly. This minimum is found to be f_{1n}, giving rise to the following Lemma.

Lemma 4.1. An oligopolistic structure with every firm operating separately is stable if and only if $f < f_{1n}(\gamma; \alpha; n)$; for all γ; α and n, where $f_{1n}(\gamma; \alpha; n)$ denotes the values of the fixed cost above which the most efficient and the least efficient firms want to join. This is the two-firm agreement that generates the highest surplus as compared with the situation in which firms operate separately.

Proof. See Appendix.

An immediate corollary of this Lemma is that when $n=3$, triopoly is stable if and only if $f < f_{13}(\gamma; \alpha)$, as we have already shown in subsection 3.2.

The economic intuition for this first result is the following: the coalition that minimizes $f_{ij} = \pi_i^n + \pi_j^n - \pi_{ij}^{n-1}$ is the one which equivalently maximizes $\pi_{ij}^{n-1} - \pi_i^n - \pi_j^n$ and so it is the most attractive for the merging parties since it generates the highest surplus to be shared; this movement is therefore the one which most "threatens" the stability of the n-firm oligopoly. The fact is that, due to the constant marginal cost assumption, the high-cost firm is shutdown and its production is transferred to the other firm: the gains are maximized when this transfer occurs from the least efficient to the most efficient firm in the market.

There is various evidence that this type of collusive agreement between very efficient and very inefficient firms may actually happen. See for example Barros(1998) for an application to a sample of concentration operations in the Portuguese industry. Brealey and Myers(1988) argue that most gains from combining complementary resources occur when small firms are acquired by large ones. They also refer to evgyefr tasr ggf o et ag firm for acquisition (see Palepu(1986)), which reinforces the importance of eliminating ine¢ciencies as a motive for merger.

This result may seem counterintuitive in that it contradicts the conventional wisdom according to which …rms would prefer good partners. Actually, according to our …ndings, in a constant marginal cost context …rms always prefer the bad partners, since they have to pay less for them and can thus keep most of the surplus generated by the agreement. The rivalry e¤ect that would induce …rms to choose closer rivals to merge with, and which is implicit in the e Cournot 1.0072 n Tc (h) Tj 6. ed TD -0.0072 Tc (u) Tj 4.32 type of behavior, is more than compensated for by this incentive to "buy" cheap ed iehe d partners. This result still holds if we consider non-constant cied

Further results on the profitability and internal stability of two-member coalitions can be found in the Appendix.

Based on the analysis of the shape of f_{ij} as a function of i and j (see Appendix) we can say that the potential external instability of the agreement $\{i; j\}$ comes from the enlargement to include either the most efficient firm left in the market (let us denote it by 1' and note that 1' is firm 1 if this does not yet belong to the group) or the least efficient firm left in the market (n'); which one is more profitable will depend on the relative efficiency of the coalition as compared with the other firms that are still operating, as shows the expression in the Appendix.

Let us denote f_{ij} by f_{ij}^{int} and the minimum of $\{f_{ij1'}; f_{ijn'}\}$ by f_{ij}^{ext} and compute $f_{ij}^{ext} - f_{ij}^{int}$. A necessary and sufficient condition for coalition $\{i; j\}$ to be stable in a range $(\alpha; f)$ is that $f_{ij}^{ext} - f_{ij}^{int}$ is positive.

The expression $f_{ij}^{ext} - f_{ij}^{int}$ is a function of $i; j; n; \gamma; \alpha$: When $\alpha = 0$ it is positive for all $n \geq 5$, so all two-member coalitions are stable in some non-empty range of f, provided the market stays with at least four (groups of) operating firms.[21] These findings can be related to Selten(1973)'s results, according to which when the number of competitors is less than or equal to 4 they have a tendency to cooperate and maximize joint profit.

For α strictly positive the difference $f_{ij}^{ext} - f_{ij}^{int}$ is concave in j and increasing most of the time (except when $f_{ij}^{ext} = f_{ij1}$ and α is too small - see Appendix). The reason is that when j is small i is small too, and the agreement does not enjoy either large external or internal stability; however as j increases internal stability is reinforced and that is the main reason for the stability of the coalition to be improved (that means, its range of stability is enlarged). As to i the evolution of $f_{ij}^{ext} - f_{ij}^{int}$ depends a lot on whether $f_{ij}^{ext} = f_{ij1}$ or $f_{ij}^{ext} = f_{ijn}$, as described in the Appendix. Besides f_{ij1} and f_{ijn}, note that a third possible expression for f_{ij}^{ext} is $f_{in(n-1)}$, which is the relevant one when the agreement already includes firm n, and i is such that the most profitable enlargement is to include firm $(n - 1)$. The function $f_{ij}^{ext} - f_{ij}^{int}$ in this case can be shown to increase with i, so the coalition with the largest range of stability will have an intermediate i, such that $f_{in}^{ext} = f_{in(n-1)}$.

There are three different candidates to be the most stable agreement with firms i and j: $\{i = 1; j = n - 1\}$, $\{i = i_1; j = n - 1\}$, where i_1 is such that $f_{ijn} = f_{ij1}$, and $\{i = i_2; j = n\}$, where i_2 is such that $f_{in(n-1)} = f_{in1}$. The most stable agreement thus involves a very inefficient firm ($j = n$ or $j = n - 1$) and either a very efficient firm ($i = 1$) or a firm with an intermediate level of efficiency. The solution will depend on the parameters of the problem ($\alpha; \gamma$ and

[21] Notice that when $\alpha = 0$ $f_{ij1'} = f_{ijn'}$:

n). The analysis carried out at the extremes of α ($\alpha = 0$ and $\alpha = \bar{\alpha}$) shows that for sufficiently low efficiency differences an intermediate i is optimal, whereas for a level of asymmetry large enough $i = 1$ is optimal.

Proposition 4.2. i) When firms are symmetric and $n \geq 5$ all two-member coalitions are stable in a well defined interval for the values of the fixed cost. For $n = 4$ or $n = 3$ no two-firm agreement can be stable.

ii) For low enough asymmetry levels the most stable two-firm agreement involves a firm i with an intermediate level of efficiency (i such that the coalition is indifferent between broadening to include the most efficient or the least efficient firm left in the market), and a firm j very inefficient ($j = n$ or $j = n - 1$). When the asymmetry level is sufficiently large, then $i = 1$ and again j very inefficient maximizes the stability of the agreement.

Proof. In the Appendix.

4.1.2. Broader agreements

In this section we discuss some of the results that can be obtained for a generic coalition with more than two elements ($r > 2$, where r denotes the cardinality of the coalition). At this level of generality results are not very conclusive.

When $r \geq 3$ the profitability condition and the condition for internal stability no longer coincide. As we said before, we restrict ourselves to the internal stability condition based on avoiding every firm from separating alone. Actually, under asymmetry it is not clear that members prefer to exit alone, it may be more attractive to leave in group (a simple example that illustrates the problem is included in the Appendix); however, for tractability reasons we confine our analysis to the simpler case, which may actually be the most relevant in a real world economy.[22]

[22] It is clear that in the symmetric case firms always prefer to deviate alone, since they then do not have to share the deviating profit with anyone else; in the asymmetric case, however, there is an additional effect, which works in the opposite direction, thus making the outcome uncertain, namely that it may be interesting to leave with the most efficient firm left in the cartel so that this new rival stays with a weaker technology. What is sure is that if the most efficient firm in the coalition wants to leave in the company of another firm it will choose the second most efficient in the coalition, if it wants to leave with two more firms it will choose the second and the third, and so on. The preference for one or another solution will depend on the size and composition of the coalition, as well as on the value of α. For $n = 3$ we are able to show that firms always prefer to deviate from the grand coalition alone rather than in the company of another firm (section 3.2 and the corresponding part of the Appendix): this result, however, is not generalizable for higher n and higher cardinality of the coalition.

Consider the coalition $C = \{e_1, e_2, \ldots, e_r\}$, with its members ranked in descending order of efficiency and $\#C = r$. Denote by f_C^{int} the condition for its internal stability:

$$\pi_C - f \geq \sum_{i \in C} \left(\pi_i|_{C\setminus\{i\}} - f\right) \;, \quad f \geq f_C^{int} = \frac{\sum_{i \in C} \pi_i|_{C\setminus\{i\}} - \pi_C}{r - 1}$$

External stability is defined either by

$$f \leq f_C^{ext1} = \pi_C + \pi_1|_C - \pi_{C\cup\{1\}} \qquad (e_1 > 1)$$

or by

$$f \leq f_C^{extn} = \pi_C + \pi_n|_C - \pi_{C\cup\{n\}} \qquad (e_r < n)$$

The profitability condition (above which the coalition forms) is given by

$$f \geq f_C^{prof} = \pi_{C\setminus\{e_f\}} + \pi_{e_f}|_{C\setminus\{e_f\}} - \pi_C$$

where e_f denotes the last firm that joined the agreement.

If coalition C is a cartel, then a necessary and sufficient condition for it to have a range of stability is that

$$f_C^{ext} - \max\{f_C^{prof}, f_C^{int}\} > 0$$

where $f_C^{ext} = \min\{f_C^{ext1}, f_C^{extn}\}$.[23] Instead, if we consider an irreversible merger, this condition is simply

$$f_C^{ext} - f_C^{prof} > 0$$

which is clearly at least as easy to verify as the condition for the stability of the cartel. Thus, whenever a cartel is stable the corresponding merger is also stable, as is patent in the figures below (note that if f_C^{ext} was not the outer line, as can happen, there would be no stability area).

If $\alpha = 0$ the two conditions for external stability are the same and the condition for internal stability is $f \geq f_C^{int} = \frac{r \cdot \pi_i|_{C\setminus\{i\}} - \pi_C}{r-1}$. It can be shown that $f_C^{ext} - f_C^{prof} > 0$ for all $r \leq n - 3$, so we conclude that symmetric mergers are

[23] We can have $f_C^{prof} \gtrless f_C^{int}$: the fact that some coalition has formed may not be sufficient to ensure that it will not be dissolved.

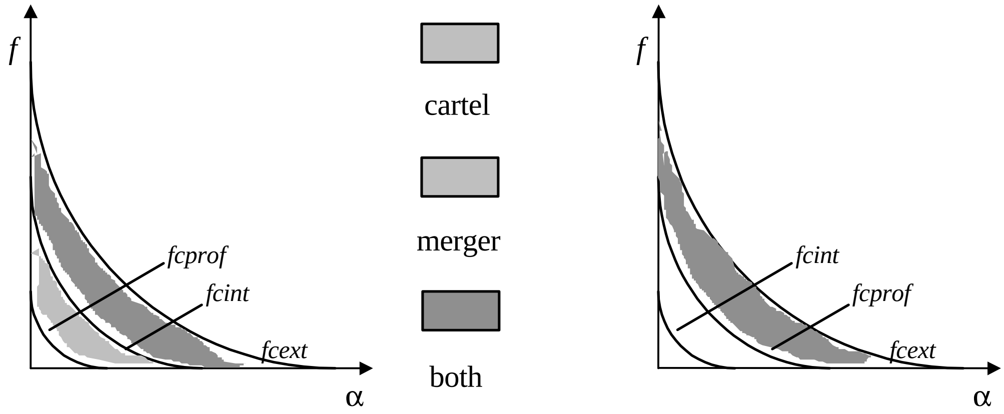


Figure 4.1: Stability areas

stable in $[f_C^{prof}; f_C^{ext}]$, provided they do not lead to triopoly, nor to duopoly.[24] Monopoly is the stable market structure for all values of the fixed cost such that $f > f_C^{ext}(r = n-3; \cdot) = f_C^{prof}(r = n-2; \cdot)$.

As to cartels, $f_C^{int} > f_C^{prof}$ for all $r > 2$ (for two-member agreements we know that $f_C^{int} = f_C^{prof}$), so a necessary and sufficient condition for the cartel with r symmetric firms to have a range of stability is that $f_C^{ext} - f_C^{int} > 0$. This condition is verified for $n \geq 5 + 4(r-2)$, or, equivalently, for $r \leq \frac{n+3}{4}$, so the higher the cardinality of the cartel, the higher has to be n. In other words, cartels which imply a too high degree of concentration cannot be stable: more than 75% of the initial number of firms must remain operating. The reason has to do with internal stability, since external stability is stronger the higher r is (as will be illustrated in the next subsection). Actually higher concentration makes it more attractive to deviate and enjoy the profits of a highly concentrated market, therefore weakening internal stability. This effect is sufficiently strong to outweigh the effect on external stability. For a given n the stability of symmetric cartels is thus more difficult the more members they enclose.

Note that the condition for the stability of symmetric cartels ($r \leq \frac{n+3}{4}$) is much more stringent than the condition for the stability of symmetric mergers ($r \leq n-3$). Mergers can thus enclose a much higher proportion of the total

[24] In the work of Espinosa and Iñarra(1995) duopolies and triopolies have a range of stability, because they consider the x-oligopoly to be externally stable whenever the (x-1)-oligopoly is not internally stable (for example, duopoly is externally stable for values of the fixed cost below which monopoly is internally unstable).

number of …rms than cartels and still be stable. The reason is that mergers do not need to verify the internal stability condition, which is more demanding the higher r is.

The analysis for ® > 0 is hardly conclusive. Based on the shape of the stability conditions, we can isolate the coalitions which are candidates to be the most stable of their type (i.e., with that number of elements), but we are not able to compare them, not even at the extremes of the range of variation of ®. As to mergers we have: fi = 1; all the other members very inef f icientg, fi = 1; all the other members very ef f icientg, or fi intermediate; all the other members very inef f icientg. For cartels there are even more possibilities, described in the Appendix.

Next we illustrate, for a particular path of aggregation, some of the results derived in this section and in the previous one.

4.2. Illustration with a speci…c collusion path

We have seen that the (n ¡ 1)-…rm oligopoly that generates the highest surplus is the one with the coalition structure {1,n}{2}{3}...{(n-1)}. The potential external inst [illegible]

If $stabm(r;:) < 0$ then C is obviously unstable, but if $stabm(r;:) > 0$ it has a range of stability.

Note that in this speci…c partition $f_C^{prof}(r+1;:) = f_C^{ext}(r;:)$. For ease of exposition $f_C^{prof}(r;:)$ will be denoted by $f_{1n(n-1)...(n-r+2)}$ and its arguments will be suppressed.[25]

It can be shown (see Appendix) that $stabm(r;:) > 0$ for all $r \leq n-5$ (at least six (groups of) operating …rms); for all $n; \gamma$; and $\alpha \in [0; \bar{\alpha}(:))$; $f \in [0; \bar{f}(:))$, i.e., that $f_{1n} < f_{1n(n-1)} < f_{1n(n-1)(n-2)} < ... < f_{1n(n-1)(n-2)...7} < f_{1n(n-1)(n-2)...76}$. This result says that as the number of coalition members increases it becomes more di¢cult to form pro…table agreements with the remaining …rms, since the surplus generated by these agreements is declining.[26] It can also be proven that the di¤erence between $f_{1n...(n-r+2)(n-r+1)}$ and $f_{1n...(n-r+2)}$ is increasing in r: the di¢culty in broadening the coalition increases as it becomes broader or, in other words, the more …rms have merged the more stable is the merger. It can be shown that $f_{1n(n-1)(n-2)...76} < f_{1n(n-1)(n-2)...765} < f_{1n(n-1)(n-2)...7654}$, but $f_{1n(n-1)(n-2)...76543} < f_{1n(n-1)(n-2)...7654}$, so, as expected, mergers which imply triopoly or duopoly are never stable. Monopoly is stable once it forms (that is, once there is an incentive to join to triopoly, i.e., for $f > f_{1n(n-1)...4}(:)$).

The following …gures illustrate for n = 8 the analysis for mergers and cartels with this speci…c collusion path.

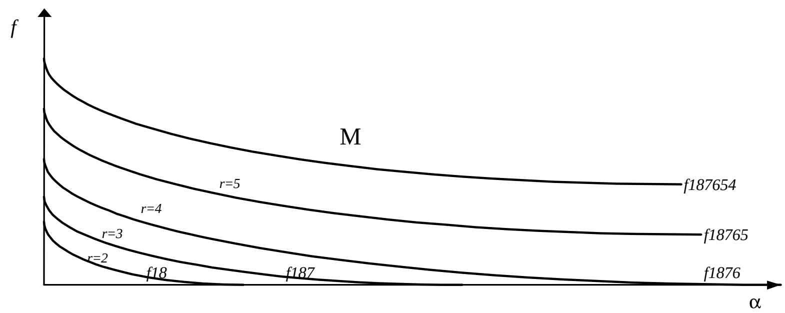


Figure 4.2: Stable mergers (n=8)

[25] The notation $f_{1n(n-1)}$, for example, means that …rms 1 and n joined …rst and then …rm (n-1) was aggregated. The order is not irrelevant, in fact $f_{1n(n-1)} \neq f_{1(n-1)n}$.

[26] Outside …rms are the more e¢cient and therefore require a higher payment. In can also be argued that the decline in the number of …rms as mergers occur increases the pro…ts of the still stand-alone …rms and hence the value that the acquiring …rm has to pay for them (so a kind of race for being the last to be acquired would take place). If the acquired …rm was paid according to the Shapley value, i.e., in accordance with its marginal contribution to the pro…t of the coalition, then this payment would increase in the degree of market concentration, as well.

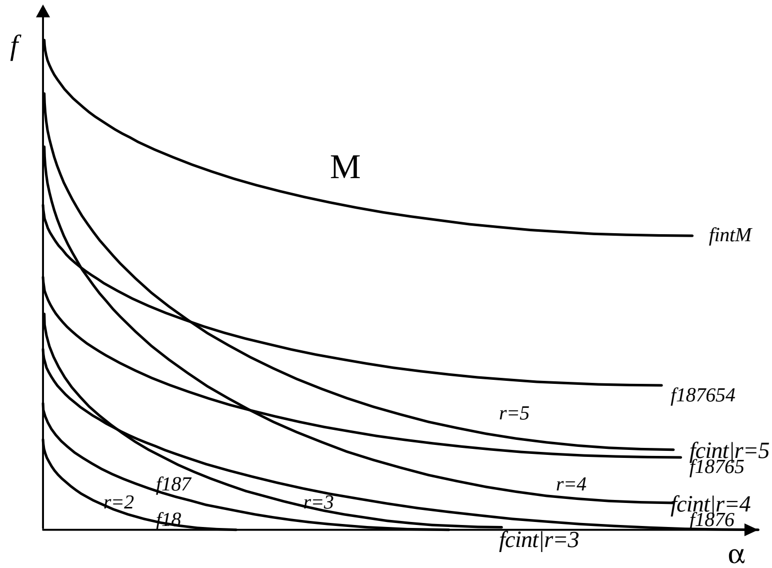


Figure 4.3: Stable cartels (n=8)

According to our previous findings, when $\alpha = 0$ the only stable cartel has just two elements. However, for $\alpha > 0$ cartels with $r \geq 3$ can be stable (provided they lead to neither triopoly, nor duopoly), since as α increases internal stability is more reinforced than external stability is weakened.

When we are dealing with cartels there can be some combinations $\{\alpha; f\}$ for which no agreement is stable, contrary to what occurs for mergers. Monopoly is not stable for all $f \geq f_{1n(n-1)\ldots 4}(\cdot)$, but only for higher f. Mergers with $r > 2$ members clearly have a larger stability range than cartels with the same number of elements. Given all the parameters of the problem either a unique stable oligopoly is determined or no stable oligopoly exists.

As is clear, the stability of an oligopolistic structure depends crucially on many parameters: the differences in efficiency between firms (α), the value of the fixed cost each firm has to pay in order to produce (f), the dimension of the market and the common marginal cost component ($\gamma = a - c$), and the number of operating firms (related to the initial number n).[27]

[27] Without complicating too much the analysis we could have considered a slightly more general demand function in which the sensitivity of quantity to price was not forced to -1. By assuming the demand $P = a - bQ$ an additional parameter would enter the analysis. With $b > 1$ (resp. $b < 1$) the quantity produced would be smaller (resp. larger) than with $b = 1$ and the price would be higher (resp. lower). Coalitions' external stability appears to be decreasing in b, which

Notice that departing from an industry totally dispersed or with a high level of dispersion we can get complete monopolization by sequential acquisition, that is, by allowing for successive rounds of mergers. This result is in line with the papers by Kamien and Zang(1990, 1991 and 1993) in which the authors obtain the impossibility of merging up to monopoly in just one round if the industry has su¢ciently numerous …rms (…rst two papers), but show that sequential acquisition makes it easier to monopolize (the 1993 paper).

In this fourth section we have been dealing with the positive aspects of the formation of coalitions between asymmetric …rms. The following section presents some results on welfare in the context of our model.

5. Welfare analysis

So far we have been dealing with the positive aspects of the formation of coalitions between asymmetric …rms. A normative analysis on the welfare consequences of these operations and its implications for antitrust policy is the subject of the present section. We shall treat the case of two-member coalitions, so it is indi¤erent if they are cartels or mergers, though we mainly use the denomination "mergers". Full proofs can be found in the Appendix.

5.1. Social welfare

In analyzing the impact of a merger on social welfare we will …rst go brie‡y through the e¤ects on the various sectors (consumers, outside …rms, and participants) and then concentrate on the overall economic impact.

When …rms are symmetric the e¤ect of a merger on consumer surplus is negative and invariant with respect to the …rms that join (it only varies with n and ³); in an asymmetric industry, too, it is negative, but we can make it softer or harder depending on the producer which is "eliminated". As expected, consumers prefer that the high-cost …rms disappear from the market, so that production is more e¢cient and thus quantity is less reduced and price less increased in the sequence of the merger. The deeper the asymmetries the more important this is.

As to the surplus of the …rms that do not participate in the merger, it can be shown that every outside …rm derives a positive bene…t from the merger of two rivals, independently of the value of ®; its production and its market share both increase.[28] E¢cient …rms are those which bene…t most and the more e¢cient is

means that the lower the sensitivity of demand to price (and the price elasticity of demand), the more attractive it is for …rms to collude and exploit the resulting market power.

[28] The no-entry assumption is important here, for the increased pro…tability of the market after the merger might trigger the entry of further …rms and thereby erode the bene…ts of the

the firm whose production ceases following the merger, the better it is for non-participants. This is a curious result for, somehow counterintuitively, it shows that mergers between "giants" do not hurt competitors - on the contrary. And the competitors who benefit most from these arrangements are exactly the closest rivals of the firms that merge. This is a consequence of our constant marginal cost assumption.

Similarly to what happens under symmetry, there are thus positive externalities for the rivals from two firms merging.[29] Notice however that it is not clear whether or not the incentive to free-ride subsists in the asymmetric context, because this will depend upon which firms merge and on how they divide the joint profit (which, remember, we do not state), as well as on the specific outside firm we are looking at. Since the global gain of non-participating firms is positive,[30] the conditional Herfindahl (the Herfindahl of non-participating firms) increases with the merger, i.e., production by non-participants becomes more concentrated (see the Appendix for the proof).

It is thus apparent that consumers and outside firms are in conflict as to the type of merger which is preferable in terms of welfare: since their interests are strictly opposed, the merger will never yield a Pareto improvement. If the sum of the variations in consumer surplus and outside firms' surplus is taken as a sufficient condition for the merger to be welfare-improving (in the spirit of Farrell and Shapiro(1990a)),[31] then $n \geq 4$ is found to be sufficient for the merger to be socially desirable in the symmetric case,[32] whereas under asymmetry a sufficient condition is $n \geq 6$. The condition for the symmetric case is equivalent to a market share of the participating firms lower than 50 per cent. With asymmetry this value is also sufficient but there are some socially desirable mergers which involve firms producing ex ante up to 60% of the total quantity placed in the market; however when the sum of the market shares exceeds 60% the merger

move.

[29] See Boyer(1992) for an analysis of mergers that harm competitors. In his model, this happens because mergers lead to a decrease in the level of marginal costs of the merged firm.

[30] Thus, according to the classification introduced in section 2, our game is one with positive spillovers. Negative spillovers arise in oligopolistic games where the formation of a coalition leads to a reduction in the production cost of member firms, making them behave more aggressively (for example, the case of research joint ventures), or, more generally, in games where the formation of agreements in the first stage improves the position of cooperating firms for the second stage of the game.

[31] Which is equivalent to assuming the private profitability of the operation, on the belief that privately unprofitable mergers will not be proposed. This condition has the considerable advantage of not requiring any information provided by the joining partners.

[32] See the more general results derived by Levin(1990) in studying "the 50-percent benchmark" and also Salant et al(1983) and Cheung(1992).

surely has negative aggregate effects. As long as the market share of the joining partners satisfies these conditions it does not matter which type of agreement we have, that is, whether it involves two firms of approximate size or two very asymmetric producers.

As we have seen above (see section 4.1) $i = 1$ and $j = n$ is the merger which maximizes internal gains, so consumers and member firms have the same preferences as to the technology of production that is going to be "abandoned". The production of the new firm is lower than the sum of the production of its components before merger and so is its market share (although higher than the market share of its largest component). The production increase of the outside firms is not sufficient, then, to compensate for the decline in participants' output, since total quantity falls. The welfare effects thus depend a lot on this reallocation of production, in the sense that the better the response of the rival firms, the less consumers suffer.

Let us consider total welfare changes and denote by

$$\Delta SW = \Delta\pi_{in} + \Delta\pi_{out} + \Delta CS$$

the (variable) change in the level of aggregate social welfare in the sequence of the merger of firms i and $j > i$; composed of the changes in participating firms' profit, in outsiders' aggregate profit and in consumer surplus (note that there is also a fixed gain corresponding to the saving in fixed cost, which will not be considered here, but that reinforces the desirability of the operation). When firms are identical a merger between two of them always hurts social welfare unless there are significant fixed cost savings.[33] The efficiency effect associated with the operation when firms are unequally efficient may render some mergers socially desirable.

Recalling that the optimal j for consumers and participating firms is $j = n$ and that the optimal j for non-participating firms is $j = 2$, the maximization of our social welfare function, which gives the same weight to all agents, will yield some j^* between 2 and n. If the weights were changed, j^* would approach the limits of this interval, accordingly. Choosing $j = 2$ never has a positive effect on social welfare. On the contrary, $j = n$ may have a positive effect if α is not too small.

There is a range of α where the change in welfare is surely negative (unless the savings in fixed cost are sufficiently high), no matter what technology "dis-

[33] This is not in contradiction with the former result of the external desirability of the merger whenever $n \geq 4$. In fact that result is based on the presumption that participants benefit from the operation, which is true only if there are significant (fixed) cost savings (see Salant et al (1983)).

appears", but as n increases this interval tends to shrink, so the higher n is, the more likely it is that the merger is socially beneficial even with no fixed cost savings. The interval corresponds (approximately) to the 28% lowest values of α when $n = 3$, 22% when $n = 4$, 9.5% when $n = 10$ (1% when $n = 100$): mergers in this range should not be allowed unless savings in fixed cost are high enough to compensate for this loss. Since we are not taking into account the fixed benefit of the merger (f), mergers which are socially good in accordance with our analysis shall be authorized with accrued reason and mergers which are "almost there" may also be permitted if f is known to be significantly different from zero.[34]

Due to the assumptions of linear demand and constant marginal costs, the social welfare variation can be written, by simple manipulation, as

$$\Delta SW = (Q^{n-1}|_{\{i,j\}})^2 \, [H^{n-1}|_{\{i,j\}} + \tfrac{1}{2}] - (Q^n)^2 \, [H^n + \tfrac{1}{2}]$$

where the terms with $\frac{1}{2}$ refer to the effect on consumer surplus and the terms with the Herfindahl index (H) refer to the effect on all firms in the industry (participants and outside firms). Since total quantity declines ($Q^{n-1}|_{\{i,j\}} < Q^n$), a necessary condition for the merger to improve social welfare is $H^{n-1}|_{\{i,j\}} > H^n$. The increase in concentration is thus necessary (though it may not be sufficient) for the overall economic effect to be positive (since consumers lose, producers must gain),[35] which shows that refusing mergers primarily on the basis of the variation induced in concentration or on the basis of established critical values for H may be seriously misleading.[36] It is good to let large firms (which is equivalent to efficient, in our model) grow, especially at the expense of small (inefficient) ones.

[34] This is clearly a partial analysis. In a general equilibrium framework the effects on all other sectors should also be taken into account.

[35] Notice that if we were increasing the number of operating firms in the industry (instead of decreasing it by merging) then the rise in H would no longer be necessary for the positive social effect, since quantity would increase (this is clearly true in the symmetric case, where an increase in n causes H to fall and SW to rise - see next footnote).

[36] This is exactly what Farrell and Shapiro(1990a) argued: when firms are symmetric, in a constant marginal cost Cournot oligopoly SW and H evolve in opposite directions; however if firms are unequally efficient there is no reason to suppose that this relationship will still hold - a decrease in the number of firms by merger may be welfare-improving (such as shifting production from high-cost to low-cost producers) and yet it causes a rise in concentration. Consequently the concentration criterion shall not be used in isolation. Daughety(1990) also claims that decreasing concentration does not imply increasing welfare and vice-versa, in a model where firms are identical but asymmetric in behavior (merged groups act as Stackelberg leaders, playing Cournot against each other, whereas the fringe plays Cournot). A similar result is obtained for them, and also for us, if a concentration ratio measure (for example C4) is used instead of H. For a wide discussion of the relationship between changes in H and changes in SW, see Farrell and Shapiro(1990b).

It can be proven that j^* does not fall below $3n/4$, so the optimal merger never involves the absorption of a low-cost firm.[37] This kind of merger induces a small deterioration in the concentration index, so although welfare and concentration both evolve in the same direction, their variations seem to follow the (correct) inverse pattern.[38] In the next subsection this relationship is discussed in more detail. For the moment let us summarize the social welfare results in the following

Proposition 5.1. When firms are not sufficiently asymmetric ($\alpha < \alpha'(n)$, with α' decreasing in n and $\alpha'(3) = 0.28\bar{\alpha}$, where $\bar{\alpha}$ is the upper limit for α such that all firms are active) the change in social welfare due to the merger of two firms is negative unless there are significant fixed cost savings associated with the operation. When the asymmetry level is large enough ($\alpha > \alpha'(n)$) the merger may be socially desirable even without fixed cost savings. The welfare maximizing operation, $\{i; j^*\}$, involves the absorption of an inefficient firm ($\frac{3}{4}n < j^* \leq n$).

Proof. In the Appendix.

5.2. Concentration and welfare

Let $H^n(\alpha;\gamma)$ stand for the pre-merger Herfindahl concentration index in our industry. The level of concentration after the merger of firms i and $j > i$ is denoted by $H^{n-1}|_{\{i;j\}}(\alpha;\gamma)$.[39] Its minimum occurs for $j = n$.

The change in the level of concentration is given by $\Delta H(\alpha;\gamma;n;j)$, which is equal to $H^{n-1}|_{\{i;j\}}(.) - H^n(.)$. As we have seen in the previous subsection $\Delta H > 0$ is a necessary condition for social welfare to increase. The minimum increase in the level of concentration is achieved, by the above argument concerning H^{n-1}, when the least efficient firm is "absorbed", which is very intuitive, because this firm is the smallest in the market. Thus, although there is an inevitable increase in concentration due to merger, the operation that maximizes social benefits ($\frac{3}{4}n < j^* \leq n$) does not imply a too serious increase in H (which is the important measure to look at), and actually the optimal mergers in both cases coincide for

[37] From a social point of view it is good to close down inefficient firms, on the one hand, and, on the other hand, to let large firms compete. This result is in accordance with the observation made by Farrell and Shapiro(1990a): "(...) it often enhances economic welfare - defined in the usual way - to close down small or inefficient firms, or, failing that, to encourage them to merge so that they produce less output. This observation may call for some rethinking of our views on policy toward competition, including horizontal merger policy" (pgs.122-3).

[38] In other words, the first derivative has the same sign for both, but second derivatives have opposite signs.

[39] It shall be calculated on the basis of the new market shares and not taking (erroneously, as we saw before) the share of the resulting firm as equal to the sum of the pre-merger shares of the participants.

most values of ®. However, for ® large the merger which maximizes the increase in social welfare ($\frac{3}{4}n < j < n$) induces an increase in concentration above the minimum (reached for $j = n$); in turn the merger which minimizes the variation in the level of concentration is not optimal with respect to social welfare bene…ts.

Proposition 5.2. The merger which minimizes the increase in concentration ($j = n$) and the merger which maximizes welfare gains ($\frac{3}{4}n < j$ · n) coincide for ® < ®$^{00}(n)$ and tend to diverge for high values of ® (®$^{00}(n)$ is decreasing in n and converges to $\frac{1}{2}$®̄, where ®̄ is the upper limit for ® such that all …rms are active).
Proof. In the Appendix.

The above results highlight the risks of an antitrust policy based simply on the analysis of concentration indexes. The higher the di¤erences among …rms in the industry, the more the welfare maximizing merger will be distinct from the one which minimizes the e¤ects on concentration.

6. Conclusion

We have analyzed the formation and stability of coalitions in an industry with asymmetric …rms and constant marginal cost where no rule of pro…t sharing is …xed and side transfers are allowed. Cartels and mergers are considered separately. They form whenever they are pro…table and for them to be stable, both internal and external stability are required for cartels, while for mergers (which are assumed to be irreversible) external stability alone is necessary. As in Rajan(1989) the emerging oligopolistic structures are characterized as a function of some market parameters: the …xed cost (as in Espinosa and Iñarra(1995)) and the e¢ciency asymmetry (to our knowledge, addressed for the …rst time). The welfare consequences of these agreements have also been studied.

The existing literature has been dealing in its majority with the formation of coalitions among identical …rms and when heterogeneity is allowed a …xed rule for dividing the joint pro…t is assumed. Our paper is an attempt to overcome both limitations, admitting asymmetry and …xing no rule of pro…t sharing.

In our model, when two …rms join, aside from the savings in …xed costs there is an e¢ciency e¤ect associated with the transfer of production from the ine¢cient to the e¢cient …rm which also works in favor of the operation. It was shown that this e¤ect dominates the rivalry e¤ect based upon which …rms would prefer to join good partners and so, contrary to what occurs when side payments are not allowed, the larger the asymmetry between merging parties, the higher the pro…tability of the agreement.

As to stability, we have proved that the most stable two-member coalition involves a very inefficient firm and either a very efficient firm or one with an intermediate level of efficiency. If we impose symmetry, then all mergers which do not imply duopoly or triopoly have an interval of stability, independently of the number of members they enclose; stable cartels, on the other hand, have to leave more than three fourths of the initial number of firms out. For asymmetric coalitions with three elements or more, only those candidates which could be the most stable could be identified, but it was not possible to compare them.

In terms of policy implications, the results suggest that policies oriented towards the spreading of technological information among firms or towards the development of better technologies of production in inefficient firms may contribute to competition on the supply side if differences between firms are kept low enough so that no agreement is profitable. Policies that result in a decrease of the fixed cost of production also promote competition.

Whether or not this is socially desirable depends upon the welfare effects of the merger and so, following the positive analysis, we have performed a normative analysis on the welfare consequences of the formation of coalitions between asymmetric firms. We have found that consumers and merging parties have the same preferences as to the optimal agreement but outside firms prefer a good technology to be "absorbed" and so, in a constant marginal cost context, mergers between "giants" are not bad news for non-participants. An initial number of firms greater than or equal to six was found to be sufficient for the social desirability of privately profitable mergers, independently of t f aense sa saal nun a nys, ir r e r fla t ia efficiency effect associated with the spreading of a better technology by merger may be sufficient to guarantee the profitability of the move, even in the absence of the other factors that have been pointed to in the literature as possible solutions to this problem, such as savings in fixed cost, differentiated product, non-linear demand and modifications in the behavior of the participants.

The subject of the endogenous determination of mergers and cartels with asymmetric firms has many limitations arising from the difficulty of the task. Our analytical framework is still simplified in that costs are linear and so is demand. However it has allowed us to identify the main problems related to this subject and to specify a mechanism for accurately determining the stable oligopolies under given realizations of the relevant parameters without need to fix the rule of division of the joint profit. This type of approach can be useful in analyzing the merging phenomenon in many industries. It may facilitate the understanding of some acquisitions and even allow some prediction. We believe that we have also provided some relevant insights for antitrust policy in asymmetric industries.

APPENDIX

A. Number of partitions

The number of k partitions (partitions in k blocks) of a set with n elements is called stirling number of the second kind and denoted $S(n;k)$. We have $S(n;k) > 0$ for all $1 \leq k \leq n$ and $S(n;k) = 0$ if $k > n \geq 1$. $S(n;k)$ can be computed according to

$$S(n;k) = \frac{1}{k!} \sum_{1 \leq i \leq k} (-1)^{k-i} \binom{k}{i} i^n$$

For example $S(n;1) = 1$; $S(n;2) = 2^{n-1} - 1$; $S(n;3) = \frac{(3^{n-1}+1)}{2} - 2^{n-1}$. This says that the number of duopolies when $n = 3$ is 3 and when $n = 4$ is 7; the number of triopolies for $n = 3$ is 1 and for $n = 4$ is 6.

There exist "triangular", "vertical" and "horizontal" recurrence relations that help in the computation of $S(n;k)$ and which can be found in any advanced combinatorics text book (see for example Comtet(1974), pgs 208-10).

For values of n and k up to considerably large limits there are double-entry built tables that immediately tell the desired number. The increase with n is more than exponential (for $n = 15$, for instance, there are 420,693,273 6-firm oligopolies).

The number of all partitions of a set with n elements is called Bell number or exponential number and is denoted by $\varpi(n)$. The following definition is clear:

$$\varpi(n) = \sum_{1 \leq k \leq n} S(n;k); \; n \geq 1$$

The recurrence relation $\varpi(n+1) = \sum_{0 \leq h \leq n} \binom{n}{h} \varpi(h); \; n \geq 0$ simplifies the computation of $\varpi(n)$ when it is not available in the previously built tables.

B. Stability analysis when n=3

With $n = 3$ there are three possible oligopolistic structures: monopoly (the grand coalition, represented either by M or by {1,2,3}), duopoly and triopoly (represented either by T or by {1}{2}{3}). There are three possible duopoly constellations: firms 1 and 2 together against firm 3 ({1,2}{3}), firms 1 and 3 together against firm 2 ({1,3}{2}) or firms 2 and 3 together competing against firm 1 ({1}{2,3}).

Variable profits in each case are

$$\pi^M = \left(\frac{a-c+\alpha}{2}\right)^2$$

$$\pi_{12}^{\{1,2\}\{3\}} = \left(\frac{(a-c)+3\alpha}{3}\right)^2; \pi_3^{\{1,2\}\{3\}} = \left(\frac{(a-c)-3\alpha}{3}\right)^2$$

$$\pi_{13}^{\{1,3\}\{2\}} = \left(\frac{(a-c)+2\alpha}{3}\right)^2; \pi_2^{\{1,3\}\{2\}} = \left(\frac{(a-c)-\alpha}{3}\right)^2$$

$$\pi_{23}^{\{1\}\{2,3\}} = \left(\frac{(a-c)-\alpha}{3}\right)^2; \pi_1^{\{1\}\{2,3\}} = \left(\frac{(a-c)+2\alpha}{3}\right)^2$$

$$\pi_1^T = \left(\frac{(a-c)+4\alpha}{4}\right)^2; \pi_2^T = \left(\frac{(a-c)}{4}\right)^2; \pi_3^T = \left(\frac{(a-c)-4\alpha}{4}\right)^2$$

Denote the difference $(a-c)$ by γ. Notice that for all firms (in whatever oligopolistic structure) to produce positive quantities (that is, to be in the market) the parameter α cannot exceed $\frac{\gamma}{4}$ (since $\alpha > \frac{\gamma}{4} \Rightarrow q_3^T = 0$) and the parameter f cannot exceed $\left(\frac{\gamma-4\alpha}{4}\right)^2$ (since $f > \left(\frac{\gamma-4\alpha}{4}\right)^2 \Rightarrow \pi_3^T - f < 0$, so q_3^T will be equal to 0). Denote these upper bounds by $\bar{\alpha}$ and $\bar{f}$, respectively, and notice that $\bar{\alpha}$ is a function of γ and $\bar{f}$ is a function of both γ and α. Let us now examine in detail the stability of the various oligopolistic structures. Recall that the grand coalition is externally stable and that triopoly is internally stable. Recall also that our stability analysis is stepwise, so the only possible movements are: from triopoly to duopoly, from duopoly either to triopoly or to monopoly, and from monopoly to duopoly.

To begin with, suppose that all firms are operating separately. Then,

• Stability of triopoly

We only have to analyze one deviation:

- two firms agree to join (external instability)

In this case the market will become a duopoly. To guarantee that this will not happen we have to assure that the following three conditions are verified:

$$\pi_1^T - f + \pi_2^T - f \geq \pi_{12}^{\{1,2\}\{3\}} - f$$
$$\pi_1^T - f + \pi_3^T - f \geq \pi_{13}^{\{1,3\}\{2\}} - f$$
$$\pi_2^T - f + \pi_3^T - f \geq \pi_{23}^{\{1\}\{2,3\}} - f$$

Solving for f we get

$$
\begin{aligned}
f &\le f_t^{\{1,2\}\{3\}}(\gamma;\alpha) = \frac{-12\alpha\gamma+\gamma^2}{72} \\
f &\le f_t^{\{1,3\}\{2\}}(\gamma;\alpha) = \frac{112\alpha^2-32\alpha\gamma+\gamma^2}{72} \\
f &\le f_t^{\{1\}\{2,3\}}(\gamma;\alpha) = \frac{64\alpha^2-20\alpha\gamma+\gamma^2}{72}
\end{aligned}
$$

where $f_t^{\{1,2\}\{3\}}(\gamma;\alpha)$, for example, is the superior limit for the stability of triopoly as regards deviations to the duopoly $\{1,2\}\{3\}$. Putting these three conditions together in a graph it appears that the binding one is $f_t^{\{1,3\}\{2\}}(\gamma;\alpha)$, and so triopoly is stable for values of α and f low enough so that $f \le f_t^{\{1,3\}\{2\}}(\gamma;\alpha)$.

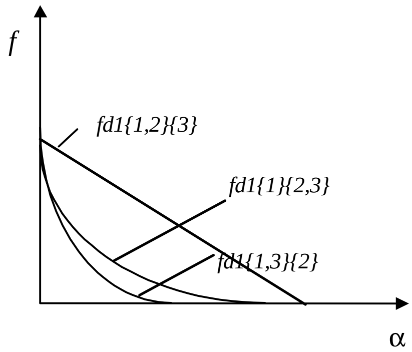


Figure B.1: Triopoly stability

- Stability of the duopolies

We have to analyze two types of deviations from duopoly:

- one of the firms which are together deviates and stays alone (internal instability, relevant only if we are considering that firms form cartels and not irreversible mergers)

In this case we will revert to triopoly. In order to avoid this we have to assure that

$$
\begin{aligned}
\pi_{(1)}^{\{1,2\}\{3\}} &\ge \pi_1^T - f \\
\pi_{(2)}^{\{1,2\}\{3\}} &\ge \pi_2^T - f \\
s.t.\ \pi_{(1)}^{\{1,2\}\{3\}} + \pi_{(2)}^{\{1,2\}\{3\}} &= \pi_{12}^{\{1,2\}\{3\}}
\end{aligned}
$$

for the duopoly $\{1,2\}\{3\}$, where $\pi_{(i)}$ denotes the unknown profit of firm i (recall that it is unknown because firms are different and we do not fix any rule of payoff division). These conditions are equivalent to

$$\pi_{12}^{\{1,2\}\{3\}} - f \geq \pi_1^T + \pi_2^T - 2f$$

Similarly we get the inequality

$$\pi_{13}^{\{1,3\}\{2\}} - f \geq \pi_1^T + \pi_3^T - 2f$$

for the duopoly $\{1,3\}\{2\}$ and the inequality

$$\pi_{23}^{\{1\}\{2,3\}} - f \geq \pi_2^T + \pi_3^T - 2f$$

for the duopoly $\{1\}\{2,3\}$.

Solving for f we get the three following conditions, which are exactly the reverse of the inequalities for the stability of triopoly vis à vis the various duopolies:[40]

$$\begin{aligned} f &\geq f_{d1}^{\{1,2\}\{3\}}(\gamma,\alpha) = f_t^{\{1,2\}\{3\}}(\gamma,\alpha) = \tfrac{-12\alpha\gamma+\gamma^2}{72} \\ f &\geq f_{d1}^{\{1,3\}\{2\}}(\gamma,\alpha) = f_t^{\{1,3\}\{2\}}(\gamma,\alpha) = \tfrac{112\alpha^2-32\alpha\gamma+\gamma^2}{72} \\ f &\geq f_{d1}^{\{1\}\{2,3\}}(\gamma,\alpha) = f_t^{\{1\}\{2,3\}}(\gamma,\alpha) = \tfrac{64\alpha^2-20\alpha\gamma+\gamma^2}{72} \end{aligned}$$

By looking again at figure B-1, it appears that the most stable duopoly regarding moves to triopoly is the one formed by firms 1 and 3 together against firm 2 (because it is stable for the largest range of α and k) and the least stable is the one formed by firms 1 and 2 together against firm 3. If the latter is stable, then all the others are, as well, but there are values of f and α for which only $\{1,3\}\{2\}$ is stable and values of f and α for which $\{1,3\}\{2\}$ and $\{1\}\{2,3\}$ are stable, but $\{1,2\}\{3\}$ is not. For $f \geq \gamma^2/72$ there is no risk of instability of any duopoly versus triopoly and the same for $\alpha \geq \gamma/12$ (since above this value all curves lie below the horizontal axis). For $\alpha < \gamma/28$ all duopolies are unstable unless f is high enough, for $\gamma/28 < \alpha < \gamma/16$ the duopoly $\{1,3\}\{2\}$ is internally stable for whatever f but the others only for f high enough, for $\gamma/16 < \alpha < \gamma/12$ duopolies $\{1,3\}\{2\}$ and $\{1\}\{2,3\}$ are internally stable for all f but the duopoly $\{1,2\}\{3\}$ is internally stable only for high f.

The intuition for these results is the following: since these conditions for stability have to do with no firm of the block having an incentive to deviate,

[40] This is due to the fact that the coalitions whose internal stability we are analyzing are made up of just two elements. For coalitions with three members or more this is no longer so.

the most internally stable duopoly is the one which entails the highest gain as compared with the deviating situation. When firms 1 and 3 join this gain is maximized, because due to our constant marginal cost assumption, all production is transferred from 3 to 1, meaning, from the least efficient to the most efficient firm in the market, and thus the efficiency effect is maximum; on the contrary when firms 1 and 2 join the gain is minimized.

- $*$ the three firms agree to move to monopoly (external instability)

This movement is always in the firms' interest, because monopoly profit is higher than the sum of duopoly profits ($\pi^M > \pi_1^{\{1\}\{2,3\}} + \pi_{23}^{\{1\}\{2,3\}} = \pi_{13}^{\{1,3\}\{2\}} + \pi_2^{\{1,3\}\{2\}} > \pi_{12}^{\{1,2\}\{3\}} + \pi_3^{\{1,2\}\{3\}}$), so duopoly is never externally stable and thus it is never stable.

- ϵ Stability of monopoly

Since we are considering separating movements by single firms, there is only one possible deviation from monopoly, which is relevant only if we are considering that firms form cartels (and not irreversible mergers, in which case monopoly is stable once it forms):

- $*$ one firm deviates alone, leaving the other two together (internal instability)

In this case we will revert to a situation of duopoly. In order to avoid this we have to assure that each firm receives in monopoly at least as much as it would receive in the deviating situation. We thus have the following conditions which guarantee that the deviation will not occur:

$$\begin{aligned}
\pi^M_{(1)} &\geq \pi_1^{\{1\}\{2,3\}} - f \\
\pi^M_{(2)} &\geq \pi_2^{\{1,3\}\{2\}} - f \\
\pi^M_{(3)} &\geq \pi_3^{\{1,2\}\{3\}} - f \\
s.t.\ \pi^M_{(1)} + \pi^M_{(2)} + \pi^M_{(3)} &= \pi^M
\end{aligned}$$

Adding the three inequalities and subtracting the fixed cost from the variable monopoly profit we get

$$\pi^M - f \geq \pi_1^{\{1\}\{2,3\}} + \pi_2^{\{1,3\}\{2\}} + \pi_3^{\{1,2\}\{3\}} - 3f$$

Solving for f we get the condition of internal stability:

$$f \geq f_m(\zeta;\alpha) = \frac{47\alpha^2 - 34\alpha\zeta + 3\zeta^2}{72}$$

If the efficiency difference happens to be larger than 0.103ζ or if the fixed cost is larger than $\zeta^2/24$, the grand coalition is stable. Higher values of ζ require higher values of f for the stability of this cartel. On the contrary, higher values of α facilitate its stability as f is decreasing in α. The economic intuition behind these results is clear: the better the demand conditions (higher ζ) the more attractive it is for a firm to separate from the others and try to make profit for itself; the more acute the differences in efficiency, the more important is the efficiency effect associated with the coalition and the easier it is to compensate the "acquired" firms (the inefficient ones).[41]

C. Results from section 4.

$P = a - Q$

n firms, n odd

$MC_1 = c - \left(\frac{n-1}{2}\right)\alpha;\ MC_2 = c - \left(\frac{n-3}{2}\right)\alpha;\ \ldots;\ MC_n = c + \left(\frac{n-1}{2}\right)\alpha$

fixed cost f

$$q_n^n = \frac{a - n(c + (\frac{n-1}{2})\alpha) + \sum_{i=1}^{n} MC_i}{n+1} = \frac{\zeta - (\frac{n^2-1}{2})\alpha}{n+1} > 0 \ , \ \alpha < \frac{2\zeta}{n^2-1} = \alpha(\zeta;n)$$

$$f < \pi_n^n = (q_n^n)^2 \ , \ f < \left[\frac{\zeta - (\frac{n^2-1}{2})\alpha}{n+1}\right]^2 = \bar{f}(\zeta;\alpha;n)$$

$\alpha(\zeta;n)$ is a vertical line in the $(\alpha;f)$ space and $\bar{f}(\zeta;\alpha;n)$ is a convex curve in the same space.

$MC_i = c - \left(\frac{n+1-2i}{2}\right)\alpha$

$f_{ij}(\zeta;\alpha;n) = \frac{1}{4n^2(n+1)^2}[2\zeta + \alpha(n^2+2n+1) - 2\alpha j(n+1)][2\zeta(n^2-2n-1) - 2\alpha j(n^3 + n^2 - n - 1) + 4\alpha in(n+1) + \alpha(n^4 - 4n^2 - 4n - 1)]$

C.1. Two-firm agreements

Proof of Lemma 1:

In this proof, such as in others below, we will abuse notation and take derivatives in order of i and j, though they are discrete variables.

f_{ij} is linearly increasing in i, so the lower i, the lower f_{ij}:

[41] For $n = 3$ it is easy to see that exiting in groups of two is less attractive than exiting alone (the stability condition in the first case lies below the stability condition in the second and actually below the horizontal axis for all α). In general this is not clear under asymmetry.

$\frac{\partial f_{ij}}{\partial i} = \frac{\alpha(\alpha+2\gamma-2\alpha j+2\alpha n-2\alpha jn+\alpha n^2)}{n(n+1)} > 0 \iff j < \frac{2\gamma+\alpha(n^2+2n+1)}{2\alpha(n+1)} \geq n$

f_{ij} is convex in j, the minimum being reached for $n-1 < j < n$:

$\frac{\partial^2 f_{ij}}{\partial j^2} = \frac{2\alpha^2(n^2-1)}{n^2} > 0$

$\frac{\partial f_{ij}}{\partial j} = 0 \iff j = j^* = \frac{2\gamma(n^2-n-1)+\alpha(n^4+n^3-2n^2-3n-1)+2\alpha ni(n+1)}{2\alpha(n-1)(n+1)^2}$

j^* is increasing in i and decreasing in α; which implies $n-1 < j^* < n$.

Therefore the minimum of f_{ij} occurs for $i=1; j=n-1$ or $i=1; j=n$. Let us compare:

$f_{1n} - f_{1(n-1)} = \frac{\alpha(\alpha n^4-2\gamma n^2-3\alpha n^2+2\gamma n+2\gamma+2\alpha)}{n^2(n+1)} < 0 \iff \alpha < \frac{2\gamma(n^2-n-2)}{n^4-3n^2+2} < \bar{\alpha}(\gamma; n)$

But for $\alpha > \frac{2\gamma(n^2-n-2)}{n^4-3n^2+2}$ $f_{1n} < 0$ and also $f_{1(n-1)} < 0$.

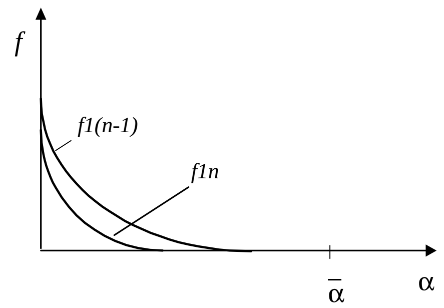


Figure C.1: Two-firm agreements

Hence $f_{1n} < f_{1(n-1)}$ in the entire range where f_{1n} and $f_{1(n-1)}$ are both positive and $\alpha < \bar{\alpha}$.

We still have to prove that $f_{1n} < \bar{f}$. Actually $\bar{f} - f_{1n}$ is concave in α and > 0 $\forall\, 0 \leq \alpha < \bar{\alpha}$, so $\bar{f} > f_{1n}$ $\forall n; \gamma; \alpha \in [0; \bar{\alpha})$. ■

Non-constant efficiency differences:

Suppose $n = 3$, $MC_1 = c - \alpha$, $MC_2 = c$, $MC_3 = c + \beta\alpha$. So far we have had $\beta = 1$; let us now consider $\beta > 1$ and $0 < \beta < 1$ and see what happens to the previous ordering $f_{13} < f_{23} < f_{12}$. The relevance of this analysis can be based, for example, on the following observation: if …rm 3 is much more ine¢cient than …rm 2 (and not just the same as …rm 2 is di¤erent from …rm 1), then does the most e¢cient …rm still prefer to join with 3? We show that if the e¢ciency e¤ect dominates the rivalry e¤ect when …rms di¤er equally, then this result is reinforced when the least e¢cient …rm becomes clearly even more ine¢cient than the others.

The …gures below illustrate the three situations. As expected, for ¯ > 1 the ordering is preserved, with f_{12} growing more and more apart from the other lines as ¯ increases (the previous dominance of the e¢ciency e¤ect over the rivalry e¤ect is reinforced), whereas f_{13} and f_{23} tend to become closer (1 and 2 are increasingly more similar in terms of "acquiring" 3 as this …rm becomes more and more ine¢cient relative to them). For 0 < ¯ < 1 the ordering can be either $f_{13} < f_{12} < f_{23}$ or $f_{13} < f_{23} < f_{12}$, depending on ¯ being < 0:75 or > 0:75, respectively; the lines f_{13} and f_{12} tend to converge as ¯ decreases, as …rms 2 and 3 become closer.

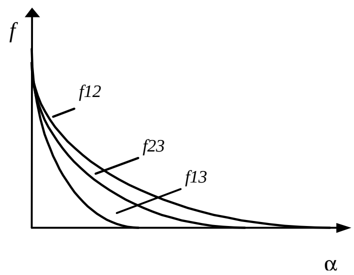


Figure C.2: Constant e¢ciency di¤erences: ¯ = 1

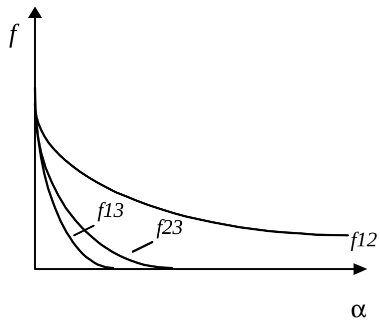


Figure C.3: Non-constant e¢ciency di¤erences: ¯ = 4

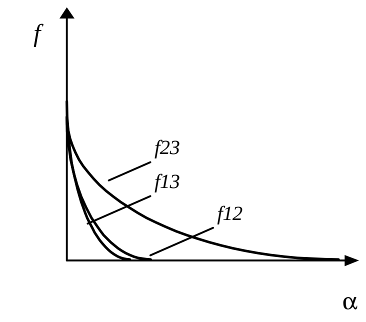


Figure C.4: Non-constant efficiency differences: $\beta = 0.1$

In any case the minimum of f_{ij} when n is equal to 3 and the efficiency differences between firms are not constant is always f_{13}. This result can be generalized for any n by induction and so we can conclude that the efficiency effect always prevails over the rivalry effect when marginal cost is constant, independently of the efficiency difference between "adjacent" firms being constant or not.

More results on the profitability and internal stability of two-member coalitions:

By maximizing $f_{ij} = \pi_i^n + \pi_j^n - \pi_{ij}^{n-1}$ in i and j we obtain the least profitable two-firm agreement, since it is the one which requires the highest values of α and f to be profitable. By performing the appropriate calculations f_{12} arises as the maximum of f_{ij} over i and j:

Given the shape of f_{ij} as a function of i and j (and given that $j > i$) the two natural candidates for the highest f_{ij} appear to be f_{12} and $f_{(n-1)n}$:

$f_{(n-1)n} - f_{12} > 0 \Leftrightarrow \alpha > \frac{2\gamma(n^2-2n-1)}{n^3+n^2-n-1} \equiv \alpha(\gamma; n)\ \forall n \geq 4$, which shows that $f_{12} > f_{(n-1)n}$ over the relevant range.

However we still have to verify that $f_{12} > f_{23}; f_{24}; \ldots; f_{2n}$ (j is larger, but i is also larger), that $f_{12} > f_{34}; f_{35}; \ldots; f_{3n}$, ..., that $f_{12} > f_{(n-2)(n-1)}$ (we have already seen that $f_{12} > f_{(n-1)n}$). But $f_{23} > f_{24} > \ldots > f_{2n}$, $f_{34} > f_{35} > \ldots > f_{3n}$, ..., so it suffices to show that $f_{12} > f_{23}$, $f_{12} > f_{34}$; ..., $f_{12} > f_{(n-2)(n-1)}$. If we can prove that $f_{i(i+1)} > f_{(i+1)(i+2)}$, then $f_{12} > f_{23}$ appears as sufficient. But $f_{12} > f_{23}$ is itself a condition of the type $f_{i(i+1)} > f_{(i+1)(i+2)}$, so we have only to prove this last inequality:

$f_{i(i+1)} - f_{(i+1)(i+2)} = \frac{\alpha(2\alpha - 2\gamma + 2\alpha i + 3\alpha n - 4\gamma n + 6\alpha i n - 3\alpha n^2 + 2\gamma n^2 + 2\alpha i n^2 - 3\alpha n^3 - 2\alpha i n^3 + \alpha n^4)}{n^2(n+1)}$, which decreases with

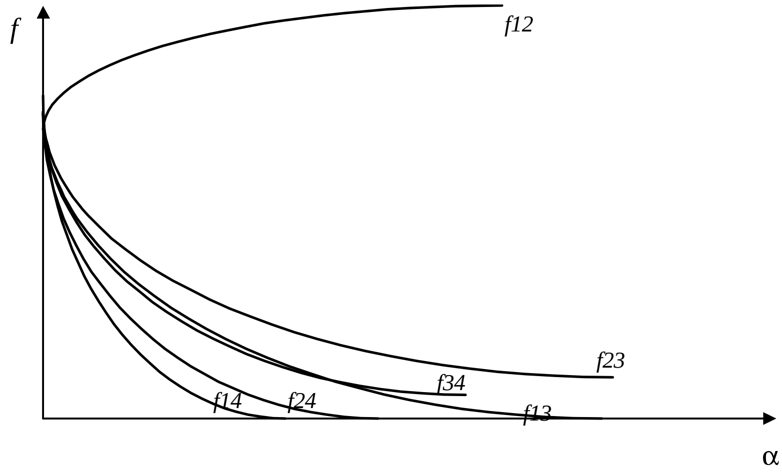


Figure C.5: Internal stability of two-firm coalitions (n=4)

$f_{ij1} - f_{ijn} = \frac{\alpha}{n(n-1)^2}(-\alpha - 2\gamma + 2\alpha j + 4\gamma n + 4\alpha in + 4\gamma in - 4\alpha jn - 4\alpha ijn - 6\gamma n^2 - \alpha in^2 - 2\gamma in^2 -$
$2\alpha i^2n^2 + 6\alpha jn^2 + 2\alpha ijn^2 - 3\alpha n^3 + 2\gamma n^3 + 2\alpha in^3 + \alpha i^2n^3 - 2\alpha jn^3 + \alpha n^4 - \alpha in^4)$

The difference $f_{ij1} - f_{ijn}$ is decreasing in j $\forall \alpha;\gamma;n; i \leq n-2$. Denote by j_1 the value of j such that $f_{ij1} = f_{ijn}$. Then

$j_1 = \frac{\alpha+2\gamma-4\gamma n-4\alpha in-4\gamma in+6\gamma n^2+\alpha in^2+2\gamma in^2+2\alpha i^2n^2+3\alpha n^3-2\gamma n^3-2\alpha in^3-\alpha i^2n^3-\alpha n^4+\alpha in^4}{2\alpha(1-2n-2in+3n^2+in^2-n^3)}$

For $j > j_1$ $f_{ij1} < f_{ijn}$, that is, the most attractive enlargement is to include firm 1, but if $j < j_1$ then $f_{ijn} < f_{ij1}$ (the most attractive enlargement is to include firm n). The choice of f_{ij1} or f_{ijn} thus depends on the relative efficiency of the coalition (i) as compared with the other firms that are still operating (all but j).

$f_{ij1} - f_{ijn}$ is convex in i and has a single root in the relevant range (the second root is $> n-2$ $\forall j;n;\gamma;\alpha$). Let us denote it by $i_1(\alpha;\gamma;n;j)$. For $i < i_1$ $f_{ijn} < f_{ij1}$ and for $i > i_1$ $f_{ij1} < f_{ijn}$.

Proof of Proposition 2:

i) $\alpha = 0 \Rightarrow f^{ext}_{ij} - f^{int}_{ij} = \frac{\gamma^2(2n^3-9n^2+3)}{n^2(n^2-1)^2} > 0$ $\forall n \geq 5$

ii) $\frac{\partial^2(f_{ij1}-f^{int}_{ij})}{\partial j^2} = \frac{\partial^2(f_{ijn}-f^{int}_{ij})}{\partial j^2} = \frac{2\alpha^2(3-6n+n^2+2n^3-n^4)}{n^2(n-1)^2} < 0$ $\forall n \geq 2$.

It can be shown that $f_{ijn} - f^{int}_{ij}$ is always increasing in j, because the value of j for which $f_{ijn} - f^{int}_{ij}$ is maximum is larger than $(n-1)$; $f_{ij1} - f^{int}_{ij}$ is also increasing if $\alpha < \alpha'(\gamma;n;i;j)$ (because the maximum occurs for $j > n$) and for

$\alpha > \alpha'$ it can decrease for j high enough ($j > j'(\alpha, \gamma, n, i)$, which can be proven to be $> n - 2$).

$\frac{\partial^2 (f_{ij1} - f_{ij}^{int})}{\partial i^2} = \frac{2\alpha^2 n(n-2)}{(n-1)^2} > 0 \ \forall n \geq 2$, so $f_{ij1} - f_{ij}^{int}$ is convex in i. It is decreasing $\forall i$, since we already know that f_{ij}^{int} increases with i $\forall i$ (see proof of Lemma 4.1) and since the minimum of f_{ij1} takes place for $i > (n-1)$, so f_{ij1} decreases in all the relevant range. The optimal i is thus i_1 such that $f_{ij1} = f_{ijn}$.

$\frac{\partial^2 (f_{ijn} - f_{ij}^{int})}{\partial i^2} = 0$, so $f_{ijn} - f_{ij}^{int}$ is linear in i. $\frac{\partial (f_{ijn} - f_{ij}^{int})}{\partial i} = \frac{1}{n(n^2-1)}[2\alpha(\alpha + 2\gamma - 2\alpha j + 2\alpha n - \alpha j n + \alpha j n^2 - \alpha n^3)]$, which can be proven to be > 0 when $\alpha < \alpha''(\gamma, n, j)$ and < 0 when $\alpha > \alpha''$. So, the optimal i can be $i = i_1$ or $i = 1$.

$\frac{\partial (f_{ijn} - f_{ij}^{int})}{\partial \alpha}|_{\alpha=0} = \frac{\gamma}{n^2(n^3 - n^2 - n + 1)}[3 - 6j + 3n - 4in + 6jn - 11n^2 + 4in^2 + 10jn^2 - 3n^3 - 8jn^3 + 6n^4 + 2jn^4 - 2n^5]$, which is increasing in i, so for values of α sufficiently low $i = i_1$ induces higher stability than $i = 1$.

$\frac{\partial (f_{ijn} - f_{ij}^{int})}{\partial \alpha}|_{\alpha=\bar{\alpha}} = \frac{2\gamma}{(n-1)^3(n^2+n^3)}[-3j + 6j^2 + 3n - 2in - 6jn + 8ijn - 12j^2 n + 2n^2 - 4in^2 + 18jn^2 - 12ijn^2 + 2j^2 n^2 - 10n^3 + 10in^3 - jn^3 + 4ijn^3 + 4j^2 n^3 + n^4 - 4in^4 - 7jn^4 - 2j^2 n^4 + 3n^5 + 3jn^5 - n^6]$, which is decreasing in i, so for values of α sufficiently close to $\bar{\alpha}$ $i = 1$ induces higher stability than $i = i_1$.

$\frac{\partial (f_{in(n-1)} - f_{in}^{int})}{\partial i} = \frac{2\alpha(\alpha + 2\gamma + \alpha n)}{n(n^2-1)} > 0 \ \forall n \geq 2$: Denote by i_2 the value of i such that $f_{in(n-1)} = f_{in1}$. The optimal i in this case is thus i_2.

Hence, the possible candidates to be the most stable two-firm agreement are: $\{i = i_2, j = n\}$, $\{i = i_1, j = n-1\}$, and $\{i = 1, j = n-1\}$. We have seen that for low α an intermediate i is optimal, whereas for α close to its upper limit $i = 1$ is optimal. $\blacksquare$

C.2. Broader agreements

Internal stability condition for coalitions with three members or more:

Consider the three-member cartel $\{i, j, l\}$, with $i < j < l$. The condition that implies that firms do not want to exit from this cartel alone is $\pi_{\{i,j,l\}} - f \geq \pi_i|_{\{j,l\}} - f + \pi_j|_{\{i,l\}} - f + \pi_l|_{\{i,j\}} - f$, $f \geq f_1(i, j, l, n, \gamma, \alpha) = \frac{\pi_i|_{\{j,l\}} + \pi_j|_{\{i,l\}} + \pi_l|_{\{i,j\}} - \pi_{\{i,j,l\}}}{2}$

On the other hand, the condition that implies that firms do not want to exit in groups of two is $2\pi_{\{i,j,l\}} - 2f \geq \pi_{\{i,j\}}|_{\{l\}} - f + \pi_{\{i,l\}}|_{\{j\}} - f + \pi_{\{j,l\}}|_{\{i\}} - f$, $f \geq f_2(i, j, l, n, \gamma, \alpha) = \pi_{\{i,j\}}|_{\{l\}} + \pi_{\{i,l\}}|_{\{j\}} + \pi_{\{j,l\}}|_{\{i\}} - 2\pi_{\{i,j,l\}}$.[43]

[43] This condition is derived from the following system:

$$\pi_{(i)}|_{\{i,j,l\}} + \pi_{(j)}|_{\{i,j,l\}} \geq \pi_{\{i,j\}}|_{\{l\}} - f$$
$$\pi_{(i)}|_{\{i,j,l\}} + \pi_{(l)}|_{\{i,j,l\}} \geq \pi_{\{i,l\}}|_{\{j\}} - f$$
$$\pi_{(j)}|_{\{i,j,l\}} + \pi_{(l)}|_{\{i,j,l\}} \geq \pi_{\{j,l\}}|_{\{i\}} - f$$
$$s.t.\ \pi_{(i)}|_{\{i,j,l\}} + \pi_{(j)}|_{\{i,j,l\}} + \pi_{(l)}|_{\{i,j,l\}} = \pi_{\{i,j,l\}}$$

So, it is not clear whether $f_1 > f_2$ or $f_2 > f_1$: the sum of the three positive terms is higher in f_2 than in f_1 and also f_2 is not being divided by 2, but the negative term is much higher in f_2 than in f_1.

$\alpha = 0$:

$$f_C^{ext1} = f_C^{extn} = \pi_C^{n-r+1} + \pi_i^{n-r+1}|_C - \pi_{C\cup\{i\}}^{n-r} = 2\left(\frac{\gamma}{n-r+2}\right)^2 - \left(\frac{\gamma}{n-r+1}\right)^2$$

$$f_C^{prof} = \pi_{C\setminus\{i\}}^{n-r+2} + \pi_i^{n-r+2}|_{C\setminus\{i\}} - \pi_C^{n-r+1} = 2\left(\frac{\gamma}{n-r+3}\right)^2 - \left(\frac{\gamma}{n-r+2}\right)^2$$

$f_C^{ext} - f_C^{prof} = \frac{\gamma^2(-17-12n+3n^2+2n^3+12r-6nr-6n^2r+3r^2+6nr^2-2r^3)}{(1+n-r)^2(2+n-r)^2(3+n-r)^2} > 0 \ \forall r \leq n-3$, since the numerator decreases in r $\forall r \leq n-2$ and is positive for $r = n-3$.

When considering mergers, monopoly is stable $\forall f > f_C^{ext}(r = n-3;\cdot) = f_C^{prof}(r = n-2;\cdot) = \frac{7\gamma^2}{400}$

$f_C^{int} = \frac{r(\pi_i^{n-r+2}|_{C\setminus\{i\}}) - \pi_C^{n-r+1}}{r-1} = \frac{r\left(\frac{\gamma}{n-r+3}\right)^2 - \left(\frac{\gamma}{n-r+2}\right)^2}{r-1}$

$f_C^{int} - f_C^{prof} = \frac{\gamma^2(r-2)(5+2n-2r)}{(2+n-r)^2(3+n-r)^2(r-1)} > 0 \ \forall r > 2$ (the fact that C forms is not sufficient to ensure that it will not be dissolved).

$f_C^{ext} - f_C^{int} = \frac{\gamma^2(27+36n+15n^2+2n^3-58r-54nr-12n^2r+39r^2+18nr^2-8r^3)}{(1+n-r)^2(2+n-r)^2(3+n-r)^2(r-1)}$. The denominator of this expression is positive $\forall r > 1$.

When $r = 2$ the numerator is equal to (apart from γ^2) $2n^3 - 9n^2 + 3$, which is $> 0 \ \forall n \geq 5$:

$r = 3 \Rightarrow$ numerator (apart from γ^2) $= 2n^3 - 21n^2 + 36n - 12 > 0 \ \forall n \geq 9$
$r = 4 \Rightarrow$ numerator (apart from γ^2) $= 2n^3 - 33n^2 + 108n - 93 > 0 \ \forall n \geq 13$
$r = 5 \Rightarrow$ numerator (apart from γ^2) $= 2n^3 - 45n^2 + 216n - 288 > 0 \ \forall n \geq 17$
$r = 6 \Rightarrow$ numerator (apart from γ^2) $= 2n^3 - 57n^2 + 360n - 645 > 0 \ \forall n \geq 21$
$r = 7 \Rightarrow$ numerator (apart from γ^2) $= 2n^3 - 69n^2 + 540n - 1212 > 0 \ \forall n \geq 25$
$r = 8 \Rightarrow$ numerator (apart from γ^2) $= 2n^3 - 81n^2 + 756n - 2037 > 0 \ \forall n \geq 29$

It seems then that as r increases by one unit, n has to increase by four units in order to preserve stability of the cartel, the minimum n required for each r being $5 + 4(r-2)$. Actually, for $n = 5 + 4(r-2)$ the numerator takes the value (apart from γ^2) $r(9r-4)$, which is always positive, while for n one unit lower ($n = 5 + 4(r-2) - 1$) the numerator is equal to $-9r^2 + 14r - 5$, which is negative $\forall r \geq 2$. The numerator of $f_C^{ext} - f_C^{int}$ is cubic in n and has only one real root, which, by the above arguing, must lie between $n = 4r-4$ and $n = 4r-3$. Taking the derivative of the numerator in order of n, one can see that it is positive $\forall n > 3r-3$ (negative otherwise), so the numerator is growing in n $\forall n \geq 5 + 4(r-2) = 4r-3$. This concludes the proof that $f_C^{ext} - f_C^{int} > 0$ $\forall n \geq 5 + 4(r-2)$.

$n \geq 5 + 4(r - 2) \Leftrightarrow r \leq \frac{n+3}{4}$. For $r = \frac{n+3}{4}$ there are $n - r + 1 = \frac{3n}{4} + \frac{1}{4}$ players in the market, so more than 75% of the initial number of firms are still operating.

$\frac{\partial f_C^{ext}}{\partial r} = \frac{2\gamma^2(-6-6n+n^3+6r-3n^2r+3nr^2-r^3)}{(1+n-r)^3(2+n-r)^3} > 0 \ \forall r \leq n-3$, because the numerator decreases in r $\forall r \leq n-2$ and is positive for $r = n-3$, so it is positive $\forall r \leq n-3$.

For monopoly $f_C^{int} = \frac{\gamma^2(4n-9)}{36(n-1)}$ (it suffices to replace r for n in the general expression for f_C^{int}).

$f_C^{prof}(r = n-1; \cdot) = \frac{\gamma^2}{72} < f_M^{int} \ \forall n \geq 3$, so f_M^{int} is the relevant condition.

$\frac{\partial f_M^{int}}{\partial n} = \frac{5\gamma^2}{36(n-1)^2} > 0$, i.e., the more members it has, the more difficult it is to sustain internal stability of the grand coalition.

$\alpha > 0$:

Consider the generic coalition $C = \{e_1, e_2, ..., e_r\}$ with members ranked in descending order of efficiency and $\# C = r$. To simplify, denote by x the marginal cost of the most efficient firm in C (e_1), by y the marginal cost of the last firm that entered the agreement (e_f, say), and by z the sum of the marginal costs of the rivals of C ($z = \sum_{i \notin C} MC_i$).

$\frac{\partial^2(f_C^{ext1} - f_C^{prof})}{\partial z^2} = \frac{\partial^2(f_C^{extn} - f_C^{prof})}{\partial z^2} = \frac{1}{(1+n-r)^2(2+n-r)^2(3+n-r)^2}[2(-17-12n+3n^2+2n^3+12r-6nr-6n^2r+3r^2+6nr^2-2r^3)]$. The denominator of this expression is always positive. As to the numerator, it is decreasing in r for all $r < n-1$ (duopoly excluded, as usual); for $r = n-2$ it is negative, but for $r = n-3$ it is positive, so it is positive $\forall r \leq n-3$ (triopoly also excluded). So $f_C^{ext} - f_C^{prof}$ is convex in the sum of the marginal costs of the outsiders $\forall r \leq n-3$.

$\frac{\partial^2(f_C^{ext1} - f_C^{prof})}{\partial x^2} = \frac{1}{(2+n-r)^2(3+n-r)^2}[2(7+18n+20n^2+8n^3+n^4-18r-40nr-24n^2r-4n^3r+20r^2+24nr^2+6n^2r^2-8r^3-4nr^3+r^4)]$. It can be shown that the numerator is decreasing in r $\forall r, n$ and positive for the highest r, so it is always positive, and hence $f_C^{ext1} - f_C^{prof}$ is convex in x.

$\frac{\partial^2(f_C^{extn} - f_C^{prof})}{\partial x^2} = \frac{2(7+32n+27n^2+6n^3-32r-54nr-18n^2r+27r^2+18nr^2-6r^3)}{(1+n-r)^2(2+n-r)^2(3+n-r)^2}$. The numerator decreases with r and is positive for the maximum r, so it is always positive, and hence $f_C^{extn} - f_C^{prof}$ is convex in the marginal cost of the most efficient firm in the agreement.

$\frac{\partial^2(f_C^{ext1} - f_C^{prof})}{\partial y^2} = \frac{\partial^2(f_C^{extn} - f_C^{prof})}{\partial y^2} = \frac{2(-5-4n-n^2+4r+2nr-r^2)}{(3+n-r)^2}$, which is $< 0 \ \forall n, r$. It is proven, then, that $f_C^{ext} - f_C^{prof}$ is concave in the marginal cost of the last firm to enter C.

Based on this analysis we can isolate the mergers which are the candidates to be the most stable, and that are mentioned in the text.

As to cartels, it can be shown that f_C^{ext} ¡ f_C^{int} is convex in the marginal cost of the most e¢cient …rm in the cartel, concave in the marginal cost of the second most e¢cient …rm in the cartel, concave in the sum of the marginal costs of the rest of the members of the cartel, and convex (concave) in the sum of the marginal costs of the outside …rms for r · $\frac{n+3}{4}$ (for r > $\frac{n+3}{4}$). The possible candidates are thus: fi intermediate; all the other members very inef f icientg, for all r, fi = 1; all the other members very ef f icientg, for r · $\frac{n+3}{4}$, and for r > $\frac{n+3}{4}$ fi = 1; all the other members intermediately ef f icientg, fi = 1; some of the other members very ef f icient and some very inef f icientg, or fi = 1; all the other members very inef f icientg. For most cartels observed r · $\frac{n+3}{4}$, so the …rst two solutions are the more relevant in practice.¥

C.3. Speci…c collusion path

stabm(r = n ¡ h; :) ¡ stabm(r = n ¡ h + 1; :) = $\frac{1}{2h^2(1+h)^2(2+h)^2(3+h)^2}$(18®^{2} + 72®³ + 72³^2 ¡ 42®^{2}h + 48®³h + 264³^{2}h ¡ 203®^{2}h^2 ¡ 248®³h^2 + 172³^{2}h^2 ¡ 262®^{2}h^3 ¡ 260®³h^3 ¡ 226®^{2}h^4 ¡ 72®³h^4 ¡ 12³^{2}h^4 ¡ 146®^{2}h^5 ¡ 4®³h^5 ¡ 58®^{2}h^6 ¡ 12®^{2}h^7 ¡ ®^{2}h^8 + 36®^{2}n + 72®³n + 24®^{2}hn + 264®³hn ¡ 124®^{2}h^{2}n + 172®³h^{2}n ¡ 130®^{2}h^{3}n ¡ 36®^{2}h^{4}n ¡ 12®³h^{4}n ¡ 2®^{2}h^{5}n + 18®^{2}n^2 + 66®^{2}hn^2 + 43®^{2}h^{2}n^2 ¡ 3®^{2}h^{4}n^2)

This expression is concave in ® 8h; ³; n and negative 8® when h ¸ 5, that is, when r · n ¡ 5. The stability area is thus increasing in r 8r · n ¡ 5 (at least six (groups of) operating …rms in the market). For the minimum r (r = 2) we have stabm(r = 2; :) = $f_{1n(n ¡ 1)}$ ¡ f_{1n} = $\frac{1}{4n^2(n^2 ¡ 1)^2}$(3®^{2} + 12®³ + 12³^2 ¡ 4®^{2}n ¡ 8®³n ¡ 19®^{2}n^2 ¡ 40®³n^2 ¡ 36³^{2}n^2 ¡ 10®^{2}n^3 ¡ 8®³n^3 + 8³^{2}n^3 + 9®^{2}n^4 + 12®³n^4 + 8®^{2}n^5 ¡ ®^{2}n^6 ¡ 2®^{2}n^7)

which is concave in ® 8n, and positive 8n ¸ 5 in the entire range 0 · ® < $\overline{®}$ where f_{1n} and $f_{1n(n ¡ 1)}$ are both > 0 (see proof of Lemma 4.1).

Hence there is a stability area when r = 2 8n ¸ 5 and so there is a stability area 8r · n ¡ 5, 8n; ³; ® 2 [0; $\overline{®}$(:)); f 2 [0; $\overline{f}$(:)) (note that r · n ¡ 5 already guarantees n ¸ 5), that is, f_{1n} < $f_{1n(n ¡ 1)}$ < $f_{1n(n ¡ 1)(n ¡ 2)}$ < ::: < $f_{1n(n ¡ 1)(n ¡ 2):::7}$ < $f_{1n(n ¡ 1)(n ¡ 2):::76}$.

We then have:

stabm(r = n ¡ 4; :) = $f_{1n(n ¡ 1)(n ¡ 2):::765}$ ¡ $f_{1n(n ¡ 1)(n ¡ 2):::76}$ =
= $\frac{®^2(37n^2 + 1614n ¡ 189523) + ®(148³n + 3228³) + 148³^2}{58800}$, which is > 0 8®; ³; n ¸ 7

stabm(r = n ¡ 3; :) = $f_{1n(n ¡ 1)(n ¡ 2):::7654}$ ¡ $f_{1n(n ¡ 1)(n ¡ 2):::765}$ =
= $\frac{®^2(7n^2 + 614n ¡ 38993) + ®(28³n + 1228³) + 28³^2}{14400}$, which is also > 0 8®; ³; n ¸ 7

$stab_m(r = n - 2;.) = f_{1n(n-1)(n-2)...76543} - f_{1n(n-1)(n-2)...7654} =$
$= \frac{\alpha^2(-13n^2+1054n-31333)+\alpha(-52\gamma n+2108\gamma)-52\gamma^2}{14400}$, which is $< 0 \ \forall \alpha, \gamma, n$, and hence $f_{1n(n-1)(n-2)...76543} < f_{1n(n-1)(n-2)...7654}$. Triopoly is not stable.

D. Results from section 5.

D.1. Social welfare

Consumer surplus

Considering the linear demand of our model, consumer surplus is given by $CS = \frac{Q^2}{2}$.

In the pre-merger situation each firm produces $q_i^n(\alpha;\gamma) = \frac{\gamma+\alpha[\frac{(n+1)(n+1-2i)}{2}]}{n+1}$;
total quantity in the market is $Q^n(\gamma) = \sum_{i=1}^{n} q_i^n = \frac{\gamma n}{n+1}$; independent of α,
and price $P^n(a;c) = a - Q^n(\gamma) = \frac{a+nc}{n+1}$. Consumer surplus is equal to $CS^n(\gamma) = \frac{(Q^n)^2}{2} = \frac{\gamma^2 n^2}{2(n+1)^2}$

$Q^n(.)$ is increasing in γ and in n. $P^n(.)$ grows with a and c and falls with n. $CS^n(.)$ varies with its arguments in the same direction as $Q^n(.)$.

$\frac{\partial Q^n(.)}{\partial n} = \frac{\gamma}{(n+1)^2} > 0$
$\frac{\partial Q^n(.)}{\partial \gamma} = \frac{n}{n+1} > 0$

Since $P^n = a - Q^n$, the proof that P^n increases with c and decreases with n is immediate. As to the parameter of exogenous demand, $\frac{\partial P^n(.)}{\partial a} = \frac{1}{n+1} > 0$. Since $CS^n = \frac{(Q^n)^2}{2}$ it is clear that the signs of the derivatives of $Q^n(.)$ apply to $CS^n(.)$ too.

If firms i and $j > i$ merge firm $l \neq i; j$ has $q_l^{n-1}|_{\{i;j\}}(\alpha;\gamma) = \frac{\gamma+\alpha[\frac{n^2+1-2j-2n(l-1)}{2}]}{n}$;
the newly created firm produces $q_i^{n-1}|_{\{i;j\}}(\alpha;\gamma) = \frac{\gamma+\alpha[\frac{n^2+1-2j-2n(i-1)}{2}]}{n}$;
total production is $Q^{n-1}|_{\{i;j\}}(\alpha;\gamma) = \frac{\gamma(n-1)}{n} - \frac{\alpha(n+1-2j)}{2n}$;
$P^{n-1}|_{\{i;j\}}(\alpha;a;c) = \frac{a+(n-1)c}{n} + \frac{\alpha(n+1-2j)}{2n}$; and
$CS^{n-1}|_{\{i;j\}}(\alpha;\gamma) = \frac{[2\gamma(n-1)-\alpha(n+1-2j)]^2}{8n^2}$.

$\Delta Q(\alpha;\gamma;n;j) = Q^{n-1}|_{\{i;j\}}(.) - Q^n(.) = -\frac{2\gamma+\alpha(n^2+2n+1)-2\alpha j(n+1)}{2n(n+1)}$. This expression is negative whenever $j < \frac{2\gamma+\alpha(n^2+2n+1)}{2\alpha(n+1)}$; which is larger than $n \ \forall \alpha < \bar{\alpha}(.)$, so $\Delta Q < 0\ \forall j;\alpha < \bar{\alpha};\gamma;n$ and $\Delta P(\alpha;\gamma;n;j) = P^{n-1}|_{\{i;j\}}(.) - P^{n}(.) = -\Delta Q > 0\ \forall j;\alpha < \bar{\alpha};\gamma;n.$[44]

$\Delta CS(\alpha;\gamma;n;j) = CS^{n-1}|_{\{i;j\}}(.) - CS^{n}(.) < 0\ \forall j;\alpha < \bar{\alpha};\gamma;n$ as an immediate consequence of $\Delta Q < 0$ and $\Delta P > 0$.

$\frac{\partial \Delta CS(.)}{\partial j} = \frac{\alpha}{2n^2}[2\gamma(n-1) - \alpha(n+1-2j)] > 0\ \forall \alpha < \frac{2\gamma(n-1)}{n+1-2j}$; which is larger than $\bar{\alpha}(.)\ \forall j$, so $\frac{\partial \Delta CS(.)}{\partial j} > 0\ \forall j;\alpha < \bar{\alpha};\gamma;n.$

$\frac{\partial^2 \Delta CS(.)}{\partial j \partial \alpha} = \frac{\gamma(n-1)-\alpha(n+1-2j)}{n^2}$, which is also $> 0\ \forall j;\alpha < \bar{\alpha};\gamma;n.$

Hence a merger between two firms always involves a decrease in consumer surplus. The more inefficient is the firm whose technology is abandoned in the sequence of the merger, the softer are the effects on the welfare of consumers (quantity is less reduced and price less increased). This effect is reinforced for higher α.

Surplus of non-participating firms

The outside firm l earns a variable profit

$\pi_l^n(\alpha;\gamma) = (q_l^n(.))^2 = \left(\frac{\gamma + \alpha[\frac{(n+1)(n+1-2l)}{2}]}{n+1}\right)^2$ before the merger and

$\pi_l^{n-1}|_{\{i;j\}}(\alpha;\gamma) = (q_l^{n-1}|_{\{i;j\}}(.))^2 = \left(\frac{\gamma + \alpha[\frac{n^2+1-2j-2n(l-1)}{2}]}{n}\right)^2$ after the merger. It derives a benefit denoted by

$\Delta\pi_l(\alpha;\gamma;n;l;j) = \pi_l^{n-1}|_{\{i;j\}}(.) - \pi_l^n(.) =$

$= \frac{(-\alpha-2\gamma+2\alpha j-2\alpha n+2\alpha jn-\alpha n^2)(-\alpha-2\gamma+2\alpha j-4\alpha n-4\gamma n+2\alpha jn+4\alpha nl-5\alpha n^2+4\alpha n^2 l-2\alpha n^3)}{4n^2(n+1)^2}$ from the operation.

Expanding $\Delta\pi_l(.)$ it becomes clear that it is convex in j, its minimum being attained at $j = \frac{\gamma}{\alpha} + \frac{n^2+2n+1-2nl}{2}$, which can be proven to be $> n\ \forall \alpha < \bar{\alpha}(.)$. So $\Delta\pi_l(.)$ is decreasing in $j\ \forall j \leq n$. For $j = n$ its value declines with l; being positive for the maximum l ($l = n-1$), so it is positive $\forall l$ and hence it is positive $\forall j;l;\alpha < \bar{\alpha};\gamma;n.$

$\frac{\partial \Delta\pi_l(.)}{\partial l} = \frac{\alpha(-\alpha-2\gamma+2\alpha j-2\alpha n+2\alpha jn-\alpha n^2)}{n(n+1)}$, which is < 0 for $j < \frac{n+1}{2} + \frac{\gamma}{\alpha(n+1)}$. But this value is larger than n for $\alpha < \bar{\alpha}(.)$, so it is clear that $\Delta\pi_l(.)$ decreases with l.

[44] This rise in price is a trivial consequence of the observation that the merger between firms i and j generates no synergies, for it only accounts for a better allocation of production across these two firms (see Proposition 2 of Farrell and Shapiro(1990a)). Significant economies of scale or learning effects are required for an oligopolistic merger to increase aggregate industry output and reduce price (again Farrell and Shapiro(1990a)).

Since we have proven that $\Delta\pi_l > 0$, it is immediate that also $\Delta q_l > 0 \ \forall l$.

$s_i^n(\alpha;\gamma) = \frac{q_i^n(\cdot)}{Q^n(\cdot)} = \frac{1}{n} + \frac{\alpha(n+1)(n+1-2i)}{2\gamma n}$, decreasing in i.

$\frac{\partial s_i^n(\cdot)}{\partial \alpha} = \frac{(n+1)(n+1-2i)}{2\gamma n} = -\frac{\gamma}{\alpha}\frac{\partial s_i^n(\cdot)}{\partial \gamma} > 0 \iff i < \frac{n+1}{2}$

The market share of any firm is an increasing function of α (decreasing function of γ) if this firm is among the more efficient (in the first half of firms as ranked according to efficiency) and diminishes with α (increases with γ) if the firm is a high-cost firm (in the second half as ranked according to efficiency).

$$s_l^{n-1}|_{\{i,j\}}(\alpha;\gamma) = \frac{q_l^{n-1}|_{\{i,j\}}}{Q^{n-1}|_{\{i,j\}}} = \frac{2\gamma+\alpha(n+1)^2-2\alpha(j+nl)}{2\gamma(n-1)-\alpha(n+1)+2\alpha j}$$

$\Delta s_l(\alpha;\gamma;n;l;j) = s_l^{n-1}|_{\{i,j\}}(\cdot) - s_l^n(\cdot) = \frac{[2\gamma+\alpha(n+1-2l)][2\gamma+\alpha(n+1)(n+1-2j)]}{2\gamma n[2\gamma(n-1)-\alpha(n+1-2j)]} > 0$ $\forall \alpha < \bar{\alpha};\gamma;n;j;l$. This is proven by showing that the first term in the numerator is positive for $\alpha < \bar{\alpha}$ and $l \leq n$, the second is positive for $\alpha < \bar{\alpha}$ and $j \leq n$, and the denominator is positive for $\alpha < \bar{\alpha}$ and $j \leq n$; so the ratio is positive.

$\frac{\partial \Delta s_l(\cdot)}{\partial l} = \frac{\alpha[2\gamma+\alpha(n+1)(n+1-2j)]}{\gamma n[-2\gamma(n-1)+\alpha(n+1-2j)]} < 0$, bearing in mind the analysis just made for $\Delta s_l(\cdot)$. This result means that the market share of the more efficient outside firms is more increased than the market share of the inefficient.

$$\Delta\pi_{out}(\alpha;\gamma;n;i;j) = \sum_{l\neq i;j} \Delta\pi_l(\alpha;\gamma;n;l;j) =$$

$= \frac{1}{4n^2(n+1)^2}[(\alpha+2\gamma-2\alpha j+2\alpha n-2\alpha jn+\alpha n^2)(-2\alpha-4\gamma+4\alpha j-7\alpha n-6\gamma n+4\alpha in+6\alpha jn-8\alpha n^2+4\gamma n^2+4\alpha in^2+2\alpha jn^2-3\alpha n^3)]$

$$\Delta\pi_{out} = \sum_{\substack{l=1\\ l\neq i;j}} (q_l^{n-1}|_{\{i;j\}})^2 - \sum_{\substack{l=1\\ l\neq i;j}} (q_l^n)^2 =$$

$$= \sum_{\substack{l=1\\ l\neq i;j}} (s_l^{n-1}|_{\{i;j\}}\cdot Q^{n-1}|_{\{i;j\}})^2 - \sum_{\substack{l=1\\ l\neq i;j}} (s_l^n \cdot Q^n)^2 =$$

$$= (Q^{n-1}|_{\{i;j\}})^2 \cdot \sum_{\substack{l=1\\ l\neq i;j}} (s_l^{n-1}|_{\{i;j\}})^2 - (Q^n)^2 \cdot \sum_{\substack{l=1\\ l\neq i;j}} (s_l^n)^2 \iff$$

$$\iff \Delta\pi_{out} = (Q^{n-1}|_{\{i;j\}})^2 \cdot H_c^{n-1} - (Q^n)^2 \cdot H_c^n$$

where H_c^n denotes the conditional Herfindahl index in the n-firm oligopoly. Bearing in mind that total quantity decreases in the sequence of the merger, so $(Q^{n-1}|_{\{i;j\}})^2 < (Q^n)^2$, and that $\Delta\pi_{out} > 0$, $H_c^{n-1} > H_c^n$.

Consumers and non-participating firms' welfare

Denote by $\Delta CS\pi_{out}(\alpha;\gamma;n;i;j) = \Delta CS(\alpha;\gamma;n;j) + \Delta\pi_{out}(\alpha;\gamma;n;i;j) = \frac{1}{8n^2(n+1)^2}(\alpha + 2\gamma - 2\alpha j + 2\alpha n - 2\alpha jn + \alpha n^2)(-3\alpha - 6\gamma + 6\alpha j - 12\alpha n - 12\gamma n + 8\alpha in + 10\alpha jn - 15\alpha n^2 + 4\gamma n^2 + 8\alpha in^2 + 4\alpha jn^2 - 6\alpha n^3)$ the variation in the welfare of consumers and outside firms in the sequence of the merger of firms i and $j > i$.

$\alpha = 0 \Rightarrow \Delta CS\pi_{out}(\gamma;n) = \frac{\gamma^2(2n^2-6n-3)}{2n^2(n+1)^2} > 0 \ \forall n \geq 4$. When $n = 3$ the merger is socially harmful.

This is the familiar result which says that in a symmetric industry with more than three firms operating at constant marginal cost and facing linear demand, antitrust authorities should not prevent the occurrence of proposed mergers (involving two firms). The fact that $\Delta CS\pi_{out}$ is negative for $n = 3$ points out the social undesirability of merging to duopoly, as also noted by Farrell and Shapiro(1990a, pg.118).

For $\alpha \neq 0$ $\Delta CS\pi_{out}(\cdot)$ is an increasing function of i : $\frac{\partial \Delta CS\pi_{out}(\cdot)}{\partial i} = \frac{1}{n(n+1)}[\alpha(\alpha + 2\gamma - 2\alpha j + 2\alpha n - 2\alpha jn + \alpha n^2)]$, which is > 0 for $j \leq n$ and $\alpha < \bar{\alpha}$.[45] It is concave in j, since $\frac{\partial^2 \Delta CS\pi_{out}(\cdot)}{\partial j^2} = -\frac{\alpha^2(2n+3)}{n^2} < 0$, so its minimum is reached for $i = 1$ and $j = 2$ or $j = n$:

$\Delta CS\pi_{out}(i = 1; j = 2; \cdot) - \Delta CS\pi_{out}(i = 1; j = n; \cdot) = \frac{1}{2n^2(n+1)}[\alpha(n-2)(-3\alpha - 6\gamma + 7\alpha n - 8\gamma n + 2\alpha n^2 + 2\gamma n^2 - 2\alpha n^3)]$, which is $< 0 \ \forall \alpha < \bar{\alpha}$ and $n \geq 7$, being null for $n = 6$. For $n = 3;4;5$ it is negative for $\alpha' = \frac{2\gamma(n^2-4n-3)}{2n^3-2n^2-7n-3} < \alpha < \bar{\alpha}$ and positive for $\alpha < \alpha'$.

The minimum of $\Delta CS\pi_{out}(\cdot)$ is thus reached for $i = 1$ and $j = 2 \ \forall n \geq 6$: with these values for i, j and n it is concave in α, being positive for $\alpha = 0$ (as expected) and for $\alpha = \bar{\alpha}$ (null for $n = 6$ and $\alpha = \bar{\alpha}$), so, due to monotonicity, we can conclude that $\Delta CS\pi_{out}(\cdot) > 0 \ \forall i; j \ \forall \alpha < \bar{\alpha}; \forall n \geq 6$.

For $n = 5$ $\Delta CS\pi_{out}(\cdot) > 0 \ \forall 0 < \alpha < \bar{\alpha} \ \forall i \geq 2; j > i$ and also when $i = 1$ and $j \geq 4$. For $i = 1; j = 3$ it is positive for α low, more precisely $\forall 0 < \alpha < \frac{17\gamma}{240}$, i.e., in 85% of the range of variation of α and $< 0 \ \forall \frac{17\gamma}{240} < \alpha < \bar{\alpha}$; for $i = 1; j = 2$ we have $\Delta CS\pi_{out}(\cdot) > 0 \ \forall 0 < \alpha < \frac{17\gamma}{318}$, which corresponds to approximately 64% of the range allowed.

[45] So the higher i, the more desirable is the merger. Notice that i is the firm which is not considered in the expression of $\Delta CS\pi_{out}$ and which is assumed to benefit from the merger. The fact that i is high means that two inefficient firms are excluded from this calculation (since $j > i$), so the value of $\Delta CS\pi_{out}$ tends to rise with i.

For n = 4 we also have ¢CS¼out(:) > 0 80 < ® < ® 8i ¸ 2; j > i and also when i = 1 and j = 4. For i = 1; j = 3 ¢CS¼out(:) > 0 80 < ® < $\frac{2^3}{37}$ (approximately 41% of the range); for i = 1; j = 2 04 range is reduced 0.0115 Tc 0.0509 Tc (=) Tj 14.16 0 T

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$s_{ij}^{n-1} - s_i^n - s_j^n = -\frac{[\gamma(n-2)-\alpha(n+1-i-j)][2\gamma+\alpha(n+1)(n+1-2j)]}{\gamma n[2\gamma(n-1)-\alpha(n+1-2j)]}$. It has already been shown (remember $\Delta s_l(\cdot)$) that the second term in the numerator and the denominator are positive for $\alpha < \bar{\alpha}$ and $j \le n$. As to the sign of the first term in the numerator, it is positive $\forall \alpha < \bar{\alpha}$; so it is clear that $s_{ij}^{n-1} < s_i^n + s_j^n$.

$s_{ij}^{n-1} - s_i^n = \frac{[2\gamma+\alpha(n+1-2i)][2\gamma+\alpha(n+1)(n+1-2j)]}{2\gamma n[2\gamma(n-1)-\alpha(n+1-2j}$

we have $j^* = n$ and since $\frac{2\gamma(2n+1)}{2n^4+2n^3-n^2-2n-1} < \frac{2\gamma(n+1)}{2n^3-n-1}$ ΔSW can be negative, as we have seen above.

$$\Delta SW = \sum_{\substack{l=1\\ l\neq j}} (q_l^{n-1}|_{\{i;j\}})^2 - \sum_{l=1} (q_l^n)^2 + \frac{(Q^{n-1}|_{\{i;j\}})^2}{2} - \frac{(Q^n)^2}{2} =$$

$$= \sum_{\substack{l=1\\ l\neq j}} (s_l^{n-1}|_{\{i;j\}}.Q^{n-1}|_{\{i;j\}})^2 - \sum_{l=1} (s_l^n.Q^n)^2 + \frac{(Q^{n-1}|_{\{i;j\}})^2}{2} - \frac{(Q^n)^2}{2} =$$

$$= (Q^{n-1}|_{\{i;j\}})^2 . \sum_{\substack{l=1\\ l\neq j}} (s_l^{n-1}|_{\{i;j\}})^2 - (Q^n)^2 . \sum_{l=1} (s_l^n)^2 + \frac{(Q^{n-1}|_{\{i;j\}})^2}{2} - \frac{(Q^n)^2}{2} \quad ,$$

$$, \quad \Delta SW = (Q^{n-1}|_{\{i;j\}})^2 . [H^{n-1} + \tfrac{1}{2}] - (Q^n)^2 . [H^n + \tfrac{1}{2}]$$

Proof of Proposition 3:

We have seen above that $\Delta SW(j = n; .)$ is negative for α lower than its first root (which is $\frac{2\gamma(2n+1)}{2n^4+2n^3-n^2-2n-1}$) and that $j^* = n$ for $\alpha \leq \frac{2\gamma(n+1)}{2n^3-n-1}$. Since $\frac{2\gamma(2n+1)}{2n^4+2n^3-n^2-2n-1} < \frac{2\gamma(n+1)}{2n^3-n-1}$ $\forall \gamma; n$, then $j^* = n$ for $\alpha < \frac{2\gamma(2n+1)}{2n^4+2n^3-n^2-2n-1}$ and so ΔSW is surely negative in that interval of α $\forall j; \gamma; n$. For $\frac{2\gamma(2n+1)}{2n^4+2n^3-n^2-2n-1} < \alpha \leq \frac{2\gamma(n+1)}{2n^3-n-1}$ j^* is still equal to n, but then the merger with j^* is socially desirable as we have seen above. Finally, for $\alpha > \frac{2\gamma(n+1)}{2n^3-n-1}$ $j^* < n$, but then $\Delta SW(j^*; .) > 0$ too, so $\Delta SW(j^*; .) > 0$ $\forall \alpha > \frac{2\gamma(2n+1)}{2n^4+2n^3-n^2-2n-1}$.

$\frac{\partial j^*}{\partial \alpha} = -\frac{\gamma(n+1)}{\alpha^2(2n^2+2n+1)} < 0$

Because j^* is decreasing in α its minimum is attained for $\alpha = \bar{\alpha}$, where it takes the value $\frac{n(3n^2+5n+2)}{4n^2+4n+2}$. Dividing it by n, observing that this ratio declines with n and then taking the limit as $n \to \infty$ we see that j^* does not fall below $\frac{3n}{4}$. ■

D.2. Concentration and welfare

$H^n(\alpha; \gamma) = \sum_{i=1} (s_i^n)^2 = \frac{1}{n} + \frac{\alpha^2(n^4+2n^3-2n-1)}{12\gamma^2 n}$

$\alpha = 0 \Rightarrow H^n = \frac{1}{n}$

As expected $H^n(.)$ is increasing in α and decreasing in γ (thus decreasing in the dimension of the market a and increasing in the marginal cost component c).

$\frac{\partial H^n(.)}{\partial \alpha} = \frac{\alpha(n-1)(n+1)^3}{6\gamma^2 n} = -\frac{\gamma}{\alpha}\frac{\partial H^n(.)}{\partial \gamma} > 0$ $\forall \alpha < \bar{\alpha}$

$H^{n-1}|_{\{i;j\}}(\alpha; \gamma) = \frac{1}{3(\alpha+2\gamma-2\alpha j+\alpha n-2\gamma n)^2}(-3\alpha^2 - 12\alpha\gamma - 12\gamma^2 + 12\alpha^2 j + 24\alpha\gamma j -$

$12\alpha^2 j^2 - 9\alpha^2 n - 12\alpha\gamma n + 12\gamma^2 n + 24\alpha^2 jn - 12\alpha^2 j^2 n - 12\alpha^2 n^2 + 24\alpha^2 jn^2 - 12\alpha^2 j^2 n^2 - 10\alpha^2 n^3 + 12\alpha^2 jn^3 - 3\alpha^2 n^4 + \alpha^2 n^5)$

$\frac{\partial H^{n-1}|_{\{i,j\}}(\cdot)}{\partial j} = \frac{4\alpha^2 n^3[\alpha(n^2-1) - 6\gamma(n+1-2j)]}{3[\alpha(n+1-2j) - 2\gamma(n-1)]}$. The denominator is $< 0\ \forall j \leq n, \gamma, n, \alpha < \bar{\alpha}$. For all $0 < \alpha < \bar{\alpha}$ the numerator decreases with j when $j \leq \frac{n-1}{2}$ and increases with j when $j \geq \frac{n-1}{2}$; so $H^{n-1}|_{\{i,j\}}$ increases with j when j is small and decreases when j is large.

In order to find the minimum of $H^{n-1}|_{\{i,j\}}(\cdot)$ let us compare $H^{n-1}|_{\{i,j\}}(j = 2, \cdot)$ with $H^{n-1}|_{\{i,j\}}(j = n, \cdot)$:

$H^{n-1}|_{\{i,j\}}(j = 2, \cdot) - H^{n-1}|_{\{i,j\}}(j = n, \cdot) = \frac{4\alpha^2 n^3 (n-2)(\alpha^2 + 16\alpha\gamma + 12\gamma^2 + \alpha^2 n - 6\alpha\gamma n + 2\alpha\gamma n^2)}{3(n-1)(2\gamma+\alpha)^2(-3\alpha + 2\gamma + \alpha n - 2\gamma n)^2}$

The long term in the numerator is convex in α and increasing $\forall\ 0 < \alpha < \bar{\alpha}$; being positive for $\alpha = 0$, so it is positive $\forall \alpha, \gamma, n$. Therefore $H^{n-1}|_{\{i,j\}}(j = 2, \cdot) > H^{n-1}|_{\{i,j\}}(j = n, \cdot)$ and so it is proven that the minimum of $H^{n-1}|_{\{i,j\}}(\cdot)$ occurs for $j = n$.

The sign of the derivative of $\Delta H(\alpha, \gamma, n, j)$ with respect to α, γ and n depends on the realizations of the parameters. It can be proven that it varies in opposite directions with α and γ.

$\frac{\partial \Delta H(\cdot)}{\partial \alpha} = -\frac{\gamma}{\alpha}\frac{\partial \Delta H(\cdot)}{\partial \gamma}$; so ΔH varies in opposite directions with α and γ.

Proof of Proposition 4:

Recall that $j^* = \frac{(n+1)[2\gamma + \alpha(2n^2+2n+1)]}{2\alpha(2n^2+2n+1)}$.

Then, as we have seen before, $j^* \leq n \iff \alpha \geq \frac{2\gamma(n+1)}{2n^3-n-1} < \bar{\alpha}$. Therefore $j^* = n$ for all $0 < \alpha \leq \frac{2\gamma(n+1)}{2n^3-n-1}$ and $j^* = \frac{(n+1)[2\gamma+\alpha(2n^2+2n+1)]}{2\alpha(2n^2+2n+1)} < n\ \forall \alpha > \frac{2\gamma(n+1)}{2n^3-n-1}$.

$\frac{\frac{2\gamma(n+1)}{2n^3-n-1}}{\bar{\alpha}} = \frac{(n+1)^2}{2n^2+2n+1}$ is decreasing in n and is equal to (approximately) 0.64 for $n = 3$, 0.61 for $n = 4$, 0.59 for $n = 5$; 0.55 for $n = 10$, 0.51 for $n = 50$. In the limit this ratio is equal to $\frac{1}{2}$, so $j^* = n$ whenever the efficiency difference is among the 50% lowest values allowed, independently of n. $\blacksquare$

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